1. **Phonons**

Consider a one-dimensional “crystal”, described by the Hamiltonian

\[ H = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left[ \pi_n^2 + (\varphi_n - \varphi_{n-1})^2 + m^2 \varphi_n^2 \right]. \]

Here \( \varphi_n \) stands for the displacement of the \( n \)th atom, at position \( x = na \) on the lattice (where the lattice spacing \( a \) has been set to one) and \( \pi_n \) is the canonically conjugate variable. The second term in \( H \) describes the coupling between nearest neighbors, while the third term is a restoring potential to the equilibrium position. The usual equal time canonical commutation relations (for the Heisenberg picture operators) apply,

\[ [\varphi_m(t), \pi_n(t)] = i\delta_{mn}. \]

In analogy with our treatment of the continuum scalar field, let us write the Fourier expansion

\[ \varphi_n = \int_{-\pi}^{\pi} \frac{dk}{(2\pi)2\omega_k} \left( a_k e^{-i\omega_k t + ikn} + a_k^\dagger e^{i\omega_k t - ikn} \right). \]

(a) What is \( \omega_k \) as a function of \( k \) and \( m \)?

(b) Why is the momentum taking values in the interval \([-\pi, \pi]\)?

(c) Derive the commutation relations for the \( a_k \) and \( a_k^\dagger \) oscillators.

(d) Write the Hamiltonian in terms of \( a_k \) and \( a_k^\dagger \). Interpret your result physically.

(e) Finally, use dimensional analysis to restore the lattice spacing \( a \) in your formulas, and take the continuum limit \( a \to 0 \).

2. Srednicki problem 22.1

3. Srednicki problem 22.2

4. Srednicki problem 22.3

5. Srednicki problem 24.3

6. Use Noether’s theorem to derive the canonical stress energy tensor \( T^{\mu\nu} \) for Maxwell theory coupled to an external conserved current, described by action

\[ S = \int d^4x \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu A_\mu \right]. \]

Is \( T^{\mu\nu} \) symmetric under exchange of the indices \( \mu \) and \( \nu \)? Is it invariant under a gauge transformation \( A_\mu \to A_\mu + \partial_\mu \lambda \)?