PHY 610 QFT, Spring 2015

Midterm Solutions

1. (a) There is a $U(1)$ symmetry acting as $\varphi_1 \mapsto e^{i\theta}\varphi_1$, $\varphi_1^* \mapsto e^{-i\theta}\varphi_1^*$, $\varphi_2 \mapsto e^{-i\theta}\varphi_2$, $\varphi_2^* \mapsto e^{i\theta}\varphi_2^*$.

   (b) Using either $j^\mu = (\partial\mathcal{L}/\partial\partial_\mu\varphi)\delta\varphi$, or varying the action under a spacetime dependent symmetry transformation $\theta(x)$, and collecting the coefficient of $\partial_\mu\theta$, we get

$$j^\mu = i\left(\varphi_1\partial^\mu\varphi_1^* - \partial^\mu\varphi_1\varphi_1^* - \varphi_2\partial^\mu\varphi_2^* + \partial^\mu\varphi_2\varphi_2^*\right).$$

   The Noether charge is

$$Q = \int d^3x\; i\left(\varphi_1\partial^0\varphi_1^* - \partial^0\varphi_1\varphi_1^* - \varphi_2\partial^0\varphi_2^* + \partial^0\varphi_2\varphi_2^*\right).$$

   Substituting the mode expansions

$$\varphi_j(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_{\mathbf{k}}}(a_j(\mathbf{k})e^{ikx} + b_j^\dagger(\mathbf{k})e^{-ikx}), \qquad j = 1,2$$

   yields

$$Q = \int \frac{d^3k}{(2\pi)^3 2\omega_{\mathbf{k}}}\;\left(a_1^\dagger(\mathbf{k})a_1(\mathbf{k}) - b_1(\mathbf{k})b_1^\dagger(\mathbf{k}) - a_2^\dagger(\mathbf{k})a_2(\mathbf{k}) + b_2(\mathbf{k})b_2^\dagger(\mathbf{k})\right),$$

   that is, the Noether charge counts the difference between the number of φ_1 and φ_2 particles (with antiparticles contributing opposite signs).

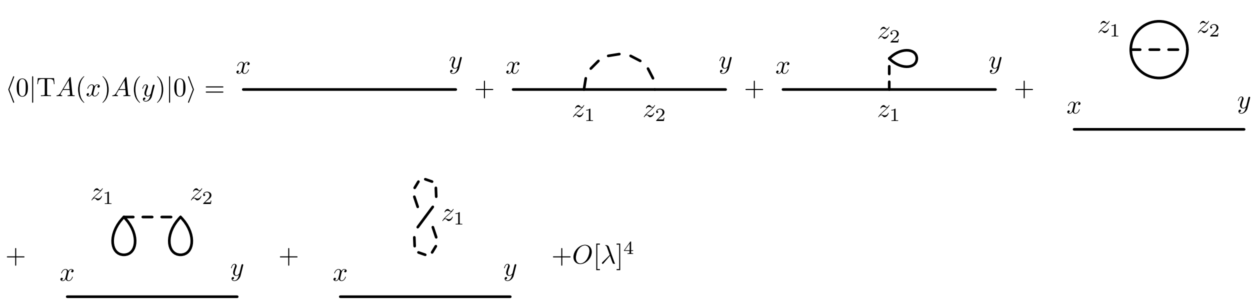
   (c) At $\mu = 0$ there is a $U(1)\times U(1)$ symmetry, acting as $\varphi_1 \mapsto e^{i\theta_1}\varphi_1$, $\varphi_2 \mapsto e^{i\theta_2}\varphi_2$.

   (d) When $\mu = 0$, $m_1 = m_2$, the symmetry is enhanced to an $SO(4)$ symmetry rotating $\varphi_j := (\Re\varphi_1, \Im\varphi_1, \Re\varphi_2, \Im\varphi_2)$. The Noether current is

$$j_a^\mu = i\partial^\mu\varphi_i(T_a)_{ij}\varphi_j,$$

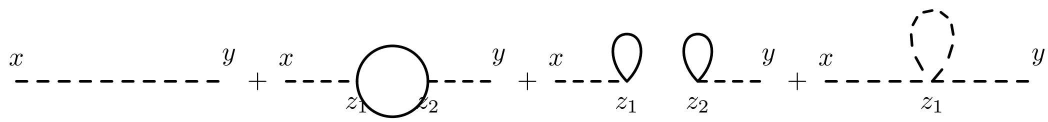
   where $(T_a)_{ij}$ are the (hermitian) generators of $SO(4)$ in the defining representation.

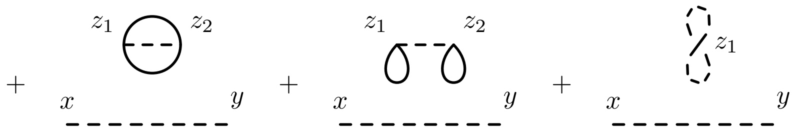
2. (a) $\langle 0|\mathrm{T}A(x)A(y)|0\rangle =$



$$= \Delta_{xy} - \lambda^2\left(\alpha^2\int d^4z_1 d^4z_2\; \Delta_{x1}\Delta_{12}\Delta_{12}\Delta_{2y} + \frac{1}{2}\Delta_{x1}\Delta_{12}\Delta_{22}\Delta_{1y} + \frac{1}{4}\Delta_{xy}\Delta_{12}^2\Delta_{12} + \frac{1}{8}\Delta_{xy}\Delta_{11}\Delta_{12}\Delta_{22} + i\beta\int d^4z_1\;\frac{1}{8}\Delta_{xy}\Delta_{11}^2\right) + O[\lambda^4]$$

$\langle 0|\mathrm{T}B(x)B(y)|0\rangle =$

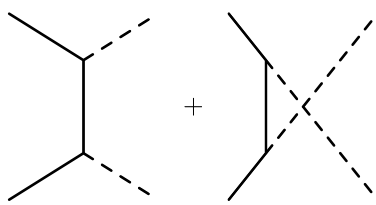




$$= \Delta_{xy} - \lambda^2 \left(\alpha^2 \int d^4z_1 d^4z_2 \, \frac{1}{2}\Delta_{x1}\Delta_{12}\Delta_{12}\Delta_{2y} + \frac{1}{4}\Delta_{x1}\Delta_{11}\Delta_{22}\Delta_{2y} + \frac{1}{4}\Delta_{xy}\Delta_{12}^2\Delta_{12} + \frac{1}{8}\Delta_{xy}\Delta_{11}\Delta_{12}\Delta_{22} + i\beta \int d^4z_1 \, \frac{1}{2}\Delta_{x1}\Delta_{11}\Delta_{1y} + \frac{1}{8}\Delta_{xy}\Delta_{11}^2 \right) + O[\lambda^4]$$

(b) i. $AA \to BB$

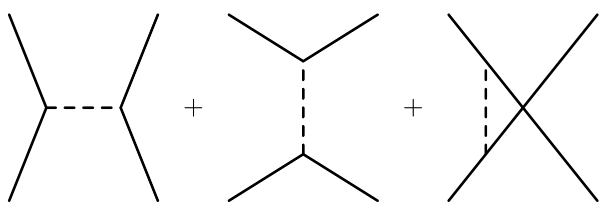
$i\mathcal{T} =$



$$= (i\alpha\lambda)^2 \left(\frac{i}{t - m^2 + i\epsilon} + \frac{i}{u - m^2 + i\epsilon} \right)$$

ii. $AA \to AA$

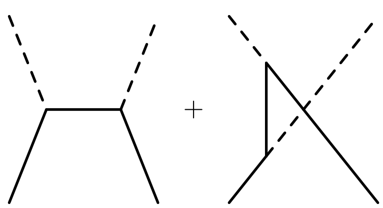
$i\mathcal{T} =$



$$= (i\alpha\lambda)^2 \left(\frac{i}{s - m^2 + i\epsilon} + \frac{i}{t - m^2 + i\epsilon} + \frac{i}{u - m^2 + i\epsilon} \right).$$

iii. $AB \to AB$

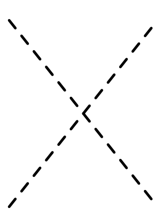
$i\mathcal{T} =$



$$= (i\alpha\lambda)^2 \left(\frac{i}{s - m^2 + i\epsilon} + \frac{i}{u - m^2 + i\epsilon} \right)$$

iv. $BB \to BB$

$i\mathcal{T} =$



$$= -i\beta\lambda^2.$$