PHY610 Midterm Exam

March 24, 2015

1. Consider the following Lagrangian density for the theory of two complex scalar fields φ_1 and φ_2,

$$\mathcal{L} = -\partial_\mu\varphi_1\partial^\mu\varphi_1^* - \partial_\mu\varphi_2\partial^\mu\varphi_2^* - m_1^2\varphi_1\varphi_1^* - m_2^2\varphi_2\varphi_2^* + \mu(\varphi_1\varphi_2 + \varphi_1^*\varphi_2^*) - \lambda(\varphi_1\varphi_1^* + \varphi_2\varphi_2^*)^2 \,.$$

   (a) What is the global symmetry of this theory, for generic values of the parameters m_1, m_2, μ and λ?

   (b) Find the Noether current associated to the global symmetry (a quick derivation or explanation will suffice). Write the Noether charge in terms of creation and annihilation operators.

   For special values of the parameters, the global symmetry is bigger:

   (c) Find the global symmetry for $\mu = 0$, $m_1 \neq m_2$.

   (d) Find the global symmetry for $\mu = 0$, $m_1 = m_2$ and write down the associated Noether current(s).

2. Consider the quantum field theory of the two real scalar fields A and B, specified by the following Lagrangian density,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{m^2}{2}A^2 - \frac{1}{2}\partial_\mu B\partial^\mu B - \frac{m^2}{2}B^2 + \frac{\alpha\lambda}{2}A^2B - \frac{\beta\lambda^2}{4!}B^4 \,.$$

   (a) You are asked to evaluate the

$$\langle 0|\mathrm{T}A(x)A(y)|0\rangle$$

   and

$$\langle 0|\mathrm{T}B(x)B(y)|0\rangle$$

   up to quadratic order in λ (that is, the terms of order $O(\lambda^n)$ with $n = 0, 1, 2$). Using position-space Feynman rules, draw to the relevant Feynman diagrams and write down the associated expressions. Use solid lines for the A propagators and dashed lines for the B propagators. Write your answer in terms of formal integral expressions involving the position-space Feynman propagator $\Delta_{xx'} = \Delta(x - x')$, which obeys

$$(-\partial_x^2 + m^2)\Delta_{xx'} = \delta^4(x)\,.$$

(Do *not* attempt to evaluate the integrals).

(b) In this part of the problem, you are asked to compute the scattering amplitude $\mathcal{T}$ to leading order in λ, for some $2 \to 2$ scattering processes. Recall that $\mathcal{T}$ is defined by

$$\langle f|i\rangle = \mathbb{I} + (2\pi)^4\delta^4(p_{in} - p_{out})i\mathcal{T}\,.$$

To compute $\mathcal{T}$ in each case, draw and evaluate the necessary momentum space Feynman diagrams. Denote the momenta of the particles as p_1 and p_2 and of the outgoing particles as p_1' and p_2', and express your answer in terms of Mandelstam invariants

$$s = -(p_1 + p_2)^2\,, \quad t = -(p_1 - p_1')^2\,, \quad u = -(p_1 - p_2')^2\,.$$

The processes for which you are asked to compute $\mathcal{T}$ are:

- $AA \to BB$
- $AA \to AA$
- $AB \to AB$
- $BB \to BB$