Lecture 1

Basic reference for this lecture: chapter 1 of Srednicki.

QFT arises from the marriage of special relativity and quantum mechanics.

I presented various arguments for the *impossibility* of single-particle relativistic quantum mechanics:

- Heuristic reasoning from $\Delta p \Delta q \sim \hbar$ and $E = mc^2$ leading to creation of particle/anti-particle pairs.
- Inconsistency of *single-particle* interpretation of the Dirac equation. I just mentioned this without any detail as we have not even introduced the Dirac equation yet.

  How inconsistencies arise in the single-particle interpretation of the Dirac equation is explained in detail in chapter 2 of Itzkykson and Zuber.

  The history of the Dirac equation and of Dirac's wildly creative resolution of the impasse are discussed for example in chapter 1 of Weinberg's book. (If you like history of physics, you could also read Schweber's great book "QED and the men who made it"). Dirac quickly realized that the single-particle interpretation of his equation was untenable and developed a multi-particle interpretation based on the "Dirac sea" and *hole theory*. While hole theory (when treated with care care) gives correct results, it is not the modern approach and is best regarded historical curiosity.

  Since after all this the single-particle interpretation of the Dirac equation is simply *wrong*, and Dirac's hole theory is morally (if not technically) wrong, I do not recommend that you spend time on these topics now. After you are comfortable with the main ideas QFT, it will be instructive to go back to them.
- Argument from causality. Again, just a sketch since we have not developed the formalism yet. The causality argument is nicely discussed in chapter 1 of Banks' book. See also section 2.1 of Peskin.

There seem to be a priori two routes towards a consistent relativistic QM:

1. Promote time T to an operator $\hat{T}$, on equal footing with the familiar position operator $\hat{X}^i$ (usually called $\hat{q}^i$). In covariant language, we have $\hat{X}^\mu(\tau) = (\hat{T}(\tau), \hat{X}^i(\tau))$, where τ is proper time.
2. Demote the position operator to an ordinary label, just like time is treated in ordinary non-relavistic QM. Both time and position are labels on operators, the *quantum fields* $\hat{\varphi}_a(x^\mu)$, which are operator-valued functions of spacetime.

Although this is at this stage far from obvious, when properly developed the two routes are completely equivalent – using one or the other viewpoint is often a matter of taste or technical convenience. By a horrible terminology, approach 1 is sometimes called the first-quantized viewpoint, and approach 2 the second-quantized viewpoint. In both cases we are just quantizing exactly *once*. Approach 1 emphasizes the role of particles, approach 2 emphasizes the role of fields. Particles are quanta of fields, and fields are coherent states of quanta. Both ideas are equally fundamental, in fact they are *dual* to each other. (Don't worry if you don't follow these vague philosophical pronouncements now, hopefully you will one day). Approach 2 is the conventional one, and the most convenient in most applications, and is the one that we will follow.

I am taking for granted a working knowledge of special relativity and of the covariant formalism (manipulations with Lorentz indices, Minkowski metric etc.). Chapter 2 of Srednicki has a very quick review. If you need to brush up on this, there are many standard textbooks. For example, most general relativity books start with a review of special relativity, see *e.g.* the first three chapters of Schutz' book "A first course in general relativity".