PHY 611 Midterm, Fall 2017

1. Sketch the RG trajectories in the m, λ plane of $\lambda\phi^4$ theory in $d = 4-\epsilon$ dimensions, where m is the mass of the scalar field. Indicate the fixed points and whether they are IR stable or UV stable. Draw arrows on the trajectories to indicate the direction of the flow. Briefly explain your reasoning.

2. Consider the position-space two-point function $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle$ of the composite operator $\mathcal{O} = [\phi^4]$ in $\lambda\phi^4$ theory, with renormalized mass fine-tuned to zero, in dimension $d \leq 4$. What is its general form? What can you say about its behavior for $|x| \to 0$ and for $|x| \to \infty$? Study in particular the cases of $d = 4$ and $d = 4 - \epsilon$ dimensions.

3. In this problem, you are asked to sketch the proof of one-loop renormalizability of spinorial QED (a massive Dirac fermion minimally coupled to an abelian gauge field). Follow these steps:
   (a) Write down the classical QED Lagrangian.
   (b) Write down the gauge-fixed Lagrangian in the covariant R_ξ gauge, (If you don't remember the precise conventions for say the definition of ξ, it's ok, so long as you are self-consistent.). At this stage, write down the ghost piece of the Lagrangian as well.
   (c) Introduce Z factors in front of each term of your gauge-fixed Lagrangian. (If you don't remember the standard conventions for the naming of the Z factors, it's ok, just use your own conventions and be consistent.)
   (d) Draw the one-loop 1PI diagrams with a non-negative superficial degree of divergence D_{sup}. Find the *actual* degree of divergence D_{real} for each of them, explaining briefly your reasoning, You can ignore the ghosts (why is this OK?).
   (e) Complete the proof of one-loop renormalizability of spinorial QED. A crucial ingredient is a certain equality between Z factors that follows from the Ward identity – which equality? You can take the requisite equality for granted, but explain briefly why it is necessary.

4. Consider a general non-abelian gauge theory with gauge potential A_a^μ, in $d = 4$ dimensions.

(a) Write down the classical action and the gauge transformations.

(b) We are going to gauge fix the action choosing the gauge-fixing functional $f_a = n_\mu A^\mu_a$, with n_μ an arbitrary fixed vector. Take the gauge-fixing piece of the Lagrangian to be

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} f_a f_a \, . \tag{1}$$

Find the ghost Lagrangian. What is the ghost propagator? and the ghost interaction vertex?