Homework 4

1. Compute the superficial degree of divergence D of a diagram with N_e external fermion lines, N_γ external photon lines and V vertices in QED, in spacetime dimension d. Repeat the exercise for scalar QED in dimension d, finding D as a function of the scalar and photon external lines and of the different kinds of vertices.

2. Srednicki problem 63.1.

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4. As an alternative to the treatment in chapter 64 of Srednicki, we can compute the magnetic moment of the electron from the effective action (64.3) by taking the non-relativistic limit of the Dirac equation coupled to an external electromagnetic field.

   (a) Write the field equation for Ψ that follows from (64.3).

   (b) Use a representation of the Dirac matrices where γ^0 is diagonal,

   $$\gamma^0 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \tag{1}$$

   In this representation, write Ψ in terms of two two-component spinors, and extract the time dependence due to the rest energy,

   $$\Psi = e^{-imt} \begin{pmatrix} \Phi \\ X \end{pmatrix} . \tag{2}$$

   Show that in the non-relativistic limit, which among other things amounts to assuming that the elecromagnetic field is small (with respect to what?), X can be solved for in terms of Φ. One obtains a Schrondiger equation for Φ,

   $$i\frac{\partial \Phi}{\partial t} = H\,\Phi\,. \tag{3}$$

   Find the Hamiltonian H. Restore powers of $\hbar$ and c by dimensional analysis. The piece of H proportional to the magnetic field $\boldsymbol{B}$ defines the magnetic moment $\boldsymbol{\mu}$,

   $$H_{\text{magn}} = -\boldsymbol{\mu}\cdot\boldsymbol{B}\,. \tag{4}$$

   You should find that magnetic moment is proportional to the electron spin,

   $$\boldsymbol{\mu} = g\frac{e}{2mc}\frac{\hbar\boldsymbol{\sigma}}{2}\,, \tag{5}$$

   where g is known as the gyromagnetic ratio.

5. Srednicki problem 67.2.