Homework 7 Solutions

Problem 1: Srednicki 88.2
From equation (75.8) we see that Neutrinos are created by $b^\dagger$ and therefore have helicity $-1/2$ while antineutrinos are created by $d^\dagger$ and therefore have helicity $+1/2$.

Problem 2: Srednicki 88.3
Fields in equation (88.33) have the symmetry $\ell_I \to e^{-i\alpha_I}\ell_I$ and $e_I \to e^{i\alpha_I}e_I$ with an independent α_I for each generation. Hence, the Dirac fields $\mathcal{E}_I$ and $\mathcal{N}_{LI}$ have charge $+1$ under the I transformation and zero under the other two.

Problem 3: Srednicki 89.4
The results of standard model 1-loop calculation are listed in Sec. 97 of Srednicki, here we copy the result. Define:

$$\mu\frac{d}{d\mu}g_i = -\frac{b_i}{16\pi^2}g_i^3 + O(g^3), \tag{1}$$

then

$$b_3 = 11 - \frac{4}{3}n, \quad b_2 = \frac{22}{3} - \frac{4}{3}n - \frac{1}{6}, \quad b_3 = -\frac{20}{9}n - \frac{1}{6}, \tag{2}$$

where $n = 3$ is the number of generations. The results follows from the general form of b for YM couple to matter:

$$b = \frac{11}{3}T(\mathrm{A}) - \frac{2}{3}T(\mathrm{R_{WF}}) - \frac{1}{3}T(\mathrm{R_{CS}}). \tag{3}$$

For unbroken $SU(3)$ part, $T(\mathrm{A}_{SU(3)}) = 3$ and $T(\mathrm{R_{WF}}) = 2$ for each generation (u & d quarks). So that:

$$b_3 = \frac{11}{3}\times 3 - \frac{2}{3}\times 2n = 11 - \frac{4}{3}n. \tag{4}$$

Similarly, for broken $SU(2)$ part, $T(\mathrm{A}_{SU(2)}) = 2$, $T(\mathrm{R_{WF}}) = 2$ for each generation (l & ν_l leptons), and $T(\mathrm{R_{CS}}) = \frac{1}{2}$ for the Higgs boson. Thus:

$$b_2 = \frac{11}{3}\times 2 - \frac{2}{3}\times 2n - \frac{1}{3}\times\frac{1}{2} = \frac{22}{3} - \frac{4}{3}n - \frac{1}{6}. \tag{5}$$

Finally, for b_3 the beta function is the same as in spinor and scalar QED, but one must use hypercharge instead of electric charge, so we have:

$$b_3 = -\frac{4}{3}\big[\frac{n}{2}(6(\frac{1}{6})^2+3(\frac{-2}{3})^2)+3(\frac{1}{3})^2)+2(\frac{-1}{2})^2+1)+\frac{1}{2}(\frac{-1}{2})^2\big] = -\frac{20}{9}n-\frac{1}{6}. \tag{6}$$

Problem 4: Srednicki 91.1
Lepton number conservation comes from the invariance under the rotations:

$$l \to e^{-i\alpha} l, \quad \bar{e} \to e^{i\alpha} \bar{e}. \tag{7}$$

One must take $\bar{\nu} \to e^{i\alpha} \bar{\nu}$ to make sure $\mathcal{L}_{\nu\text{Yuk}}$ is invariant, but then the neutrino mass term breaks the invariance.