Homework 6 Solutions

Problem 1 : (Srednicki problem 84.1)
a) The potential is

$$V = \frac{1}{2}m^2v^2\sum_i a_i^2 + \frac{1}{4}v^4\left(\lambda_1\sum_i a_i^4 + \lambda_2\left(\sum_i a_i^2\right)^2\right). \tag{1}$$

Solving the equation $\frac{\partial V}{\partial v} = 0$ for v and plugging it back into V one gets

$$V = \frac{-\frac{1}{4}m^4}{\lambda_1 A(\alpha) + \lambda_2 B(\alpha)}, \tag{2}$$

where $A(\alpha) = \sum_i a_i^4$ and $B(\alpha) = 1$.

b) From equation (1) we see that in order for V to be bounded from below

$$\lambda_1 A(\alpha) + \lambda_2 B(\alpha) > 0. \tag{3}$$

c) Equation (2) implies that V reaches it's minimum when the denominator is minimum (note the minus sign in the numerator).

d) Here we need to minimize the function

$$f(a_i,\alpha,\beta) = \sum_i\left(\frac{1}{4}a_i^4 + \frac{1}{2}a_i^2 + \beta a_i\right) \tag{4}$$

For the a_i'a we get the cubic equation

$$a_i^3 + \alpha a_i + \beta = 0. \tag{5}$$

Hence, there are three different solutions whose sum is equal to zero (since the coefficient of a_i^2 is zero).

e)Starting from the identity $\sum_i(x_i - \bar{x})^2 \geq 0$ which further implies $\sum_i x_i^2 \geq N\bar{x}^2$, when $x_i = a_i^2$ we have that

$$\sum_i \alpha^4 \geq \frac{1}{N}\left(\sum_i a_i^2\right)^2 = \frac{1}{N}. \tag{6}$$

For even N it is easy to see that $a_i = \pm\frac{1}{\sqrt{N}}$ with $N_+ = N_- = \frac{1}{2}N$. For odd N a little bit more complicated analysis shows that again $a_i = \pm\frac{1}{\sqrt{N}}$ with $N_\pm = \frac{1}{2}(N \pm 1)$.

Problem 2 : (Srednicki problem 87.1)
From equation (88.15) we see that

$$Q = T^3 + Y. \tag{7}$$

Problem 3 : (Srednicki problem 87.4)
The Feynamn rules for the Lagrangian (87.27) are ($r = -p - q$):

$$\begin{aligned}
iV^{\mu\nu\rho}_{WW\gamma}(p,q,r) &= -ie[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^n g^{\rho\mu}],\\
iV^{\mu\nu\rho}_{WWZ}(p,q,r) &= -ie\cot\theta_W[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^n g^{\rho\mu}],\\
iV^{\mu\nu\rho\sigma}_{\gamma\gamma WW} &= -ie^2(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}),\\
iV^{\mu\nu\rho\sigma}_{\gamma ZWW} &= -ie^2\cot\theta_W(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}),\\
iV^{\mu\nu\rho\sigma}_{ZZWW} &= -ie^2\cot^2\theta_W(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}),\\
iV^{\mu\nu\rho\sigma}_{WWWW} &= i\frac{e^2}{\sin^2\theta_W}(2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}),\\
iV^{\mu\nu}_{HWW} &= -2i\frac{M_W^2}{v}g^{\mu\nu},\\
iV^{\mu\nu}_{HZZ} &= -2i\frac{M_Z^2}{v}g^{\mu\nu},\\
iV^{\mu\nu}_{HHWW} &= -2i\frac{M_W^2}{v^2}g^{\mu\nu},\\
iV^{\mu\nu}_{HHZZ} &= -2i\frac{M_Z^2}{v^2}g^{\mu\nu},\\
iV_{3H} &= -3i\frac{m_H^2}{v},\\
iV4H &= -3i\frac{m_H^2}{v^2}.
\end{aligned}$$