Problem 1: (Srednicki problem 84.1)

a) The potential is

\[ V = \frac{1}{2} m^2 v^2 \sum_i a_i^2 + \frac{1}{4} v^4 \left( \lambda_1 \sum_i a_i^4 + \lambda_2 \left( \sum_i a_i^2 \right)^2 \right). \]  

Solving the equation \( \frac{\partial V}{\partial v} = 0 \) for \( v \) and plugging it back into \( V \) one gets

\[ V = -\frac{1}{4} m^4 \frac{A(\alpha)}{\lambda_1 A(\alpha) + \lambda_2 B(\alpha)}, \]  

where \( A(\alpha) = \sum_i a_i^4 \) and \( B(\alpha) = 1 \).

b) From equation (1) we see that in order for \( V \) to be bounded from below

\[ \lambda_1 A(\alpha) + \lambda_2 B(\alpha) > 0. \]  

Equation (2) implies that \( V \) reaches it’s minimum when the denominator is minimum (note the minus sign in the numerator).

d) Here we need to minimize the function

\[ f(a_i, \alpha, \beta) = \sum_i \left( \frac{1}{4} a_i^4 + \frac{1}{2} a_i^2 + \beta a_i \right) \]  

For the \( a_i \)'s we get the cubic equation

\[ a_i^3 + \alpha a_i + \beta = 0. \]  

Hence, there are three different solutions whose sum is equal to zero (since the coefficient of \( a_i^2 \) is zero).

e) Starting from the identity \( \sum_i (x_i - \bar{x})^2 \geq 0 \) which further implies \( \sum_i x_i^2 \geq N \bar{x}^2 \), when \( x_i = a_i^2 \) we have that

\[ \sum_i a_i^4 \geq \frac{1}{N} \left( \sum_i a_i^2 \right)^2 = \frac{1}{N}. \]  

For even \( N \) it is easy to see that \( a_i = \pm \frac{1}{\sqrt{N}} \) with \( N_+ = N_- = \frac{1}{2} N \). For odd \( N \) a little bit more complicated analysis shows that again \( a_i = \pm \frac{1}{\sqrt{N}} \) with \( N_+ = \frac{1}{2}(N + 1) \).
Problem 2: (Srednicki problem 87.1)
From equation (88.15) we see that
\[ Q = T^3 + Y. \] (7)

Problem 3: (Srednicki problem 87.4)
The Feynman rules for the Lagrangian (87.27) are \((r = -p - q)\):

\[
\begin{align*}
    iV_{\gamma WW}^{\mu \nu} (p,q,r) &= -ie[(p - q)\gamma^\mu g^{\nu\rho} + (q - r)\gamma^\rho g^{\mu\nu} + (r - p)\gamma^\sigma g^{\mu\rho}], \\
    iV_{\gamma WZ}^{\mu \nu} (p,q,r) &= -ie \cot \theta_W [(p - q)\gamma^\mu g^{\nu\rho} + (q - r)\gamma^\rho g^{\mu\nu} + (r - p)\gamma^\sigma g^{\mu\rho}], \\
    iV_{\gamma WWW}^{\mu \nu \rho \sigma} &= -ie^2(2g^{\mu\rho}g^{\sigma\rho} - g^{\mu\rho}g^{\sigma\rho} - g^{\mu\sigma}g^{\rho\rho}), \\
    iV_{\gamma WWZ}^{\mu \nu \rho \sigma} &= -ie^2\cot^2 \theta_W (2g^{\mu\rho}g^{\sigma\rho} - g^{\mu\rho}g^{\sigma\rho} - g^{\mu\sigma}g^{\rho\rho}), \\
    iV_{WWW}^{\mu \nu \rho \sigma} &= -ie^2\cot \theta_W (2g^{\mu\rho}g^{\sigma\rho} - g^{\mu\rho}g^{\sigma\rho} - g^{\mu\sigma}g^{\rho\rho}), \\
    iV_{WWZ}^{\mu \nu \rho \sigma} &= -ie^2\cot^2 \theta_W (2g^{\mu\rho}g^{\sigma\rho} - g^{\mu\rho}g^{\sigma\rho} - g^{\mu\sigma}g^{\rho\rho}), \\
    iV_{WZZ}^{\mu \nu \rho \sigma} &= -ie^2\cot \theta_W (2g^{\mu\rho}g^{\sigma\rho} - g^{\mu\rho}g^{\sigma\rho} - g^{\mu\sigma}g^{\rho\rho}), \\
    iV_{WW}^{\mu \nu} &= -2i\frac{M_W^2}{v} g^{\mu\nu}, \\
    iV_{WWZ}^{\mu \nu} &= -2i\frac{M_Z^2}{v} g^{\mu\nu}, \\
    iV_{WWW}^{\mu \nu} &= -2i\frac{M_W v}{v^2} g^{\mu\nu}, \\
    iV_{WZZ}^{\mu \nu} &= -2i\frac{M_Z v}{v^2} g^{\mu\nu}, \\
    iV_{HWW}^{\mu \nu} &= -3i\frac{m_H^2}{v} g^{\mu\nu}, \\
    iV_{HZZ}^{\mu \nu} &= -3i\frac{m_H v}{v^2} g^{\mu\nu}, \\
    iV_{HWW}^{\mu \nu} &= -3i\frac{m_H^2}{v^2} g^{\mu\nu}.
\end{align*}
\]