Homework 4

1. Below is a list of QFT models in various spacetime dimensions (look at the measure $d^d x$ to know the value of the dimension d in each case). For each model, answer these questions (briefly explain your reasoning):

   - Does the model contain non-renormalizable (irrelevant) terms?
   - If the answer is no, is the model renormalizable by power-counting as written, or is it missing terms needed to cancel UV divergences?
   - List the additional terms (if any) and draw representative diagrams contributing to the corresponding counterterms to leading loop order.
   - What is the degree of divergence of those diagrams?

   [For simplicity of notation, Z factors are omitted in the Lagrangians below. In other terms, all parameters and fields are *bare* quantities, so they should carry a subscript $_0$, *e.g* $\lambda_0\phi_0^4$ etc., but I omitted the subscript to avoid cluttering.]

   (a)
   $$S=\int d^4x\left(-\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i-\frac{\lambda}{4}(\phi_i\phi_i)^2\right),\quad i=1,\dots N$$

   (b)
   $$S=\int d^4x\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}M^2\phi^2+\bar{\Psi}(i\partial\!\!\!/-m)\Psi+g\phi\bar{\Psi}\gamma_5\Psi\right)$$

   (c)
   $$S=\int d^4x\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}M^2\phi^2+\bar{\Psi}(i\partial\!\!\!/-m)\Psi+g\phi\bar{\Psi}\Psi\right)$$

   (d)
   $$S=\int d^3x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-(D^\mu\phi)^\dagger D_\mu\phi-m^2\phi^\dagger\phi-\frac{g}{4}(\phi^\dagger\phi)^2\right),\quad D_\mu=\partial_\mu-ieA_\mu$$

2. Srednicki problem 78.1

3. Srednicki problem 73.1