Homework 4

1. Below is a list of QFT models in various spacetime dimensions (look at the measure $d^d x$ to know the value of the dimension $d$ in each case). For each model, answer these questions (briefly explain your reasoning):

- Does the model contain non-renormalizable (irrelevant) terms?
- If the answer is no, is the model renormalizable by power-counting as written, or is it missing terms needed to cancel UV divergences?
- List the additional terms (if any) and draw representative diagrams contributing to the corresponding counterterms to leading loop order.
- What is the degree of divergence of those diagrams?

[For simplicity of notation, $Z$ factors are omitted in the Lagrangians below. In other terms, all parameters and fields are bare quantities, so they should carry a subscript $0$, e.g. $\lambda_0 \phi_0^4$ etc., but I omitted the subscript to avoid cluttering.]

(a) 
$$ S = \int d^4 x \left( -\frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i - \frac{\lambda}{4} (\phi_i \phi_i)^2 \right), \quad i = 1, \ldots N $$

(b) 
$$ S = \int d^4 x \left( -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} M^2 \phi^2 + \bar{\Psi} (i \partial - m) \Psi + g \phi \bar{\Psi} \gamma_5 \Psi \right) $$

(c) 
$$ S = \int d^4 x \left( -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} M^2 \phi^2 + \bar{\Psi} (i \partial - m) \Psi + g \phi \bar{\Psi} \Psi \right) $$

(d) 
$$ S = \int d^3 x \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - (D^\mu \phi)^\dagger D_\mu \phi - m^2 \phi \phi^\dagger + \frac{g}{4} (\phi \phi^\dagger)^2 \right), \quad D_\mu = \partial_\mu - i e A_\mu $$

2. Srednicki problem 78.1

3. Srednicki problem 73.1