Homework 3 Solutions

Problem 1 : (Srednicki problem 62.2)
Considering the R_ξ gauge the only thing that changes is the extra term in the photos propagator. First, we note that the self-energy diagram of the photon does not include a photos propagator and therefore Z_3 doesn't change. For the fermion self-energy diagram the contribution of the ξ-term in the photon propagator is

$$i\Delta\Sigma = (\xi - 1)e^2 \int \frac{d^d l}{(2\pi)^d} \frac{\not{l}(\not{p} - \not{l} + m)\not{l}}{((p+l)^2 + m^2)\, l^4} - i(\Delta Z_2)\not{p} - i(\Delta Z_m)m. \tag{1}$$

This can be evaluated in the usual way and one can get

$$Z_2 = 1 - \xi \frac{e^2}{8\pi^2\epsilon}, \tag{2}$$

and

$$Z_m = 1 - (\xi + 3)\frac{e^2}{8\pi^2\epsilon}. \tag{3}$$

Similarly for the vertex correction we get an extra contribution

$$i\Delta V_\mu = (\xi - 1)e^3 \int \frac{d^l}{(2\pi)^4} \frac{\not{l}(-\not{l} + m)\gamma_\mu(-\not{l} + m)\not{l}}{\left(l^2 + m^2\right)^2 l^4} + ie(\Delta Z_1)\gamma_\mu, \tag{4}$$

and again one can get

$$Z_1 = 1 - \xi \frac{e^2}{8\pi^2\epsilon}. \tag{5}$$

Setting $\xi = 0$ we recover the Z-factors in the Lorenz gauge.

Problem 2 : (Srednicki problem 67.2)
Replacing ε_1' by k_1' and using momentum conservation the amplitude becomes

$$\mathcal{T} = e^2 \bar{v}_2 \left[\not{\varepsilon}_2' \left(\frac{-\not{p}_1 + \not{k}_1' + m}{m^2 - t} \right) \not{k}_1' + \not{k}_1' \left(\frac{-\not{p}_2 - \not{k}_1' + m}{m^2 - u} \right) \not{\varepsilon}_2' \right] u_1. \tag{6}$$

Using $\not{k}_1'\not{k}_1' = -k_1^2 = 0$ we have

$$\mathcal{T} = e^2 \bar{v}_2 \left[\not{\varepsilon}_2' \left(\frac{-\not{p}_1 + m}{m^2 - t} \right) \not{k}_1' + \not{k}_1' \left(\frac{-\not{p}_2 + m}{m^2 - u} \right) \not{\varepsilon}_2' \right] u_1. \tag{7}$$

Then we can commmute the momenta by using

$$(-\not{p}_1 + m)\not{k}_1' = \not{k}_1'(\not{p}_1 + m) + 2p_1 \cdot k_1, \tag{8}$$

and

$$\not{k}_1'(-\not{p}_2+m) = (\not{p}_2+m)\not{k}_1' - 2p_2\cdot k_1, \tag{9}$$

and noting that $(\not{p}_1+m)u_1 = 0$ and $\bar{v}_2(-\not{p}_2+m) = 0$ we arrive at

$$\mathcal{T} = e^2\bar{v}_2\left[\not{\epsilon}_2'\left(\frac{2p_1\cdot k_1}{m^2-t}\right)\not{k}_1' - \not{k}_1'\left(\frac{2p_2\cdot k_1}{m^2-u}\right)\not{\epsilon}_2'\right]u_1. \tag{10}$$

It's easy to see now that this vanishes since $2p_1\cdot k_1 = t-m^2$ and $2p_12\cdot k_1 = u-m^2$.

Problem 3 : (Srednicki problem 70.6)
Since the gauge field is in the adjoint representation of the gauge group, the covariant derivative acting on the field strength is equal to

$$D_\rho F^a_{\mu\nu} = \partial_\rho F^a_{\mu\nu} - gf^{abc}A^b_\rho F^c_{\mu\nu}. \tag{11}$$

Adding three cyclically permuted covariant derivatives we get

$$\begin{aligned}&D_\mu F^a_{\nu\rho} + D_\nu F^a_{\rho\mu} + D_\rho F^a_{\mu\nu}\\ =&\partial_\nu F^a_{\nu\rho} + \partial_\nu F^a_{\rho\mu} + \partial_\rho F^a_{\mu\nu} - gf^{abc}A^b_\mu F^c_{\nu\rho} - gf^{abc}A^b_\nu F^c_{\rho\mu} - gf^{abc}A^b_\rho F^c_{\mu\nu}\end{aligned} \tag{12}$$

Using the definition of the field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \tag{13}$$

most of the terms cancel and we end up with

$$\begin{aligned}D_\mu F^a_{\nu\rho} + D_\nu F^a_{\rho\mu} + D_\rho F^a_{\mu\nu} &= -gA^c_\mu A^d_\nu A^e_\rho\left(f^{acd}f^{bde} + f^{adb}f^{bce} + f^{eab}f^{cdb}\right)\\ &= 0\end{aligned} \tag{14}$$

which is zero by the Jacobi Identity (70.4).

Problem 4 : (Srednicki problem 74.1)
From the free field expansion of the gauge field we have that

$$-i\int d^3x e^{ikx}\overleftrightarrow{\partial_0}A_\mu = \sum_\lambda \epsilon^\mu_\lambda(k)a^\dagger_\lambda(k). \tag{15}$$

Contracting this equation with ck_μ we get

$$-c\sqrt{2}\omega a^\dagger_<(k) = -c\xi\{Q, b^\dagger(k)\}, \tag{16}$$

which is the BRST transformation of

$$|\chi\rangle = -c\xi b^\dagger(k)\,|\psi\rangle\,. \tag{17}$$

Problem 5 : (Weinberg chapter 15 problem 2)
In this problem, instead of the usual gauge condition we have to use

$$f^a(x) = \nabla_i A_i^a(x). \tag{18}$$

However, the calculation is exactly the same with the only difference that the Faddev-Popov determinant will be

$$det[-\nabla_i D_i^{ab} \delta^4(x-y)]. \tag{19}$$

Therefore the ghost action is

$$\mathcal{L}_{\text{ghost}} = \bar{c^a} \nabla^2 c^a - g f^{abc} \bar{c^a} \nabla_i A_i^b c^c. \tag{20}$$

From this expression we can easily read the ghost propagator which is

$$\tilde{\Delta}^{ab}(k) = \frac{1}{\mathbf{k}^2} \delta^{ab}. \tag{21}$$