QFT for critical phenomena and Wilsonian ideas

Some references:

- Cardy's book "Scaling and renormalization in statistical physics" (already mentioned in suggested books on the webpage)
- E. Brezin's book, "Introduction of Statistical Field Theory"
- Chapters 12 and 13 of Peskin's book.
- Chapter 23 of Schwartz's book.
- Polchinski's paper "Renormalization and effective Lagrangians", pdf linked from the webpage. Required reading: introduction and toy model of section 2.

Critical exponents versus conformal dimensions "cheat sheet". We are going to relate the critical exponents α, β, γ, δ, η and ν to the conformal dimensions Δ_ϕ and Δ_{ϕ^2}. This analysis applies to systems with a two-dimensional phase diagram, *i.e.*, with two relevant scalar operators, ϕ and ϕ^2. We use the terminology appropriate to magnetic systems: ϕ couples to the magnetic field h and ϕ^2 couples to $m^2 \sim t$, where t is the reduced temperature, $t = (T - T_c)/T_c$. We indicate by f the free energy per unit volume, which has mass dimension d.

- Exponent η:

  Two-point function of the fundamental scalar

  $$\langle \phi(x)\phi(0)\rangle = \frac{1}{|x|^{2\Delta_\phi}} \qquad \Delta_\phi = \frac{d}{2} - 1 + \gamma_\phi\,,$$

  where γ_ϕ is the "anomalous dimension" of ϕ, *i.e.* the deviation from the naive engineering dimension. Comparing with

  $$\langle \phi(x)\phi(0)\rangle = \frac{1}{|x|^{d-2+\eta}}$$

  we find

  $$\eta = 2\gamma_\phi\,.$$

- Exponent ν:

  Defined from the behavior of the correlation length,

  $$\xi \sim \frac{1}{|T - T_c|^\nu}\,.$$

Dimensional analysis gives

$$\xi \sim \frac{1}{(m^2)^{\frac{1}{[m^2]}}} \sim \frac{1}{t^{\frac{1}{[m^2]}}} \, .$$

The composite operator ϕ^2 has dimension $\Delta_{\phi^2} = 2(\frac{d}{2} - 1) + \gamma_{\phi^2}$. Since ϕ^2 appears as $m^2\phi^2$ in the action, we have

$$[m^2] + \Delta_{\phi^2} = d$$

which implies

$$[m^2] = 2 - \gamma_{\phi^2} \, , \quad [m] = 1 - \frac{1}{2}\gamma_{\phi^2} \, .$$

This gives

$$\nu = \frac{1}{[m^2]} = \frac{1}{2 - \gamma_{\phi^2}} \, .$$

Note also that $\gamma_{\phi^2} = 2\gamma_m$. Indeed γ_m (in Srednicki's conventions) is defined by the RG equation

$$\frac{d}{d \log \mu} \log m(\mu) = \gamma_m \, ,$$

which implies that the dimension of m is shifted from 1 to $[m] = 1 - \gamma_m$.

- Exponent α:

$$\frac{\partial^2 f}{\partial t^2} \sim |t|^{-\alpha}$$

Dimensional analysis gives

$$\alpha = -\frac{d - 2[m^2]}{[m^2]} = 2 - \frac{d}{2 - \gamma_{\phi^2}}$$

- Exponent β:

$$\lim_{h \to 0^+} \langle \phi \rangle \sim (-t)^{\beta} \, .$$

Dimensional analysis gives

$$\beta = \frac{\Delta_\phi}{[m^2]} = \frac{d/2 - 1 + \gamma_\phi}{2 - \gamma_{\phi^2}} \, .$$

- Exponent γ:

$$\frac{\partial^2 f}{\partial h^2} \sim |t|^{-\gamma}$$

Dimensional analysis gives

$$\gamma = -\frac{d - 2[h]}{[m^2]} = \frac{2(d - \Delta_\phi) - d}{2 - \gamma_{\phi^2}} = \frac{2 - 2\gamma_\phi}{2 - \gamma_{\phi^2}}$$

- Exponent δ:

$$\langle\phi\rangle \sim h^{1/\delta}$$

  Dimensional analysis gives

$$\delta = \frac{[h]}{\Delta_\phi} = \frac{d/2 + 1 - \gamma_\phi}{d/2 - 1 + \gamma_\phi}\,.$$

The “scaling relations”

$$\gamma = \nu(2-\eta)\,, \quad \alpha + 2\beta + \gamma = 2\,, \quad \delta = 1 + \frac{\gamma}{\beta}\,, \quad \alpha = 2 - \nu d$$

are automatically obeyed.