Homework 6

1. Srednicki problem 11.2
2. Srednicki problem 11.3
3. (From the 2015 midterm). Consider the following Lagrangian density for the theory of two complex scalar fields φ_1 and φ_2,

$$\mathcal{L} = -\partial_\mu\varphi_1\partial^\mu\varphi_1^* - \partial_\mu\varphi_2\partial^\mu\varphi_2^* - m_1^2\varphi_1\varphi_1^* - m_2^2\varphi_2\varphi_2^* + \mu(\varphi_1\varphi_2 + \varphi_1^*\varphi_2^*) - \lambda(\varphi_1\varphi_1^* + \varphi_2\varphi_2^*)^2 .$$

   (a) What is the global symmetry of this theory, for generic values of the parameters m_1, m_2, μ and λ?

   (b) Find the Noether current associated to the global symmetry (a quick derivation or explanation will suffice). Write the Noether charge in terms of creation and annihilation operators.

   For special values of the parameters, the global symmetry is bigger:

   (c) Find the global symmetry for $\mu = 0$, $m_1 \neq m_2$.

   (d) Find the global symmetry for $\mu = 0$, $m_1 = m_2$ and write down the associated Noether current(s).

4. (From the 2015 midterm). Consider the quantum field theory of the two real scalar fields A and B, specified by the following Lagrangian density,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{m^2}{2}A^2 - \frac{1}{2}\partial_\mu B\partial^\mu B - \frac{m^2}{2}B^2 + \frac{\alpha\lambda}{2}A^2 B - \frac{\beta\lambda^2}{4!}B^4 .$$

   You are asked to evaluate the connected part of

$$\langle 0|\mathrm{T}A(x)A(y)|0\rangle$$

   and of

$$\langle 0|\mathrm{T}B(x)B(y)|0\rangle$$

   up to quadratic order in λ (that is, the terms of order $O(\lambda^n)$ with $n = 0, 1, 2$). Using position-space Feynman rules, draw the relevant Feynman diagrams and write down the associated expressions. Use solid lines for the A propagators and dashed lines for the B propagators. Write your answer in terms of formal integral expressions involving the position-space Feynman propagator $\Delta_{xx'} = \Delta(x - x')$, which obeys

$$(-\partial_x^2 + m^2)\Delta_{xx'} = \delta^4(x - x') .$$

   Do *not* attempt to evaluate the integrals.