PHY 610 QFT, Spring 2017

HW5 Solutions

1. Since φ is complex, its e^{ikx} and e^{-ikx} modes are independent. In terms of creation and annihilation operators, φ includes outgoing a-particles and incoming b-particles. Likewise, $\varphi^\dagger$ includes incoming a-particles and outgoing b-particles. Now use the fact that J sources $\varphi^\dagger$ and $J^\dagger$ sources φ. Charge arrows pointing toward the vertex point away from J sources while charge arrows pointing away from the vertex point toward $J^\dagger$ sources. Thus incoming a implies incoming charge and outgoing a implies outgoing charge. The opposite is true for b. Of course this is all convention dependent, but we use the conventions from homework 3 where this theory was seen before.

   Since momentum and charge flow direction agree for a, we will use "particles" to describe a-particles and "anti-particles" to describe b-particles in the Feynman rules:

   - Label ingoing particles and outgoing antiparticles with a charge arrow toward the vertex.
   - Label outgoing particles and infoing antiparticles with a charge arrow away from the vertex.
   - Connect internal lines according to the rule that the only allowed vertex has two arrows of each direction.
   - Label all lines with a momentum and conserve it at each vertex.
   - Diagram value is $-i\lambda$ for each vertex and $\frac{-i}{k^2+m^2-i\epsilon}$ for each internal line having 4-momentum k.

   If we wish to reduce notation, a mnemonic is labelling an external antiparticle with minus its momentum. This will allow charge arrows to double as momentum arrows as long as we remember to account for this when solving for the momentum that an internal line should have.

2. Since $\mathcal{L}_1 = g\chi\varphi\varphi^\dagger$ contains exactly one field of each type, it already has the conventional suppression factor. This means the vertex factor is simply ig.

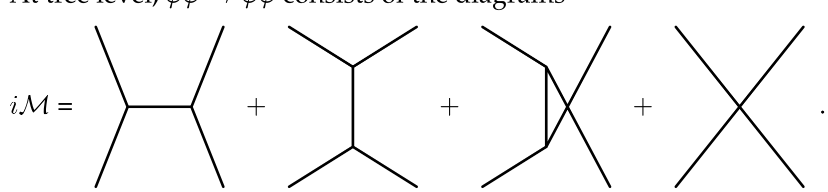
3. Redefining $\varphi \mapsto \varphi + \lambda\varphi^2$ in free field theory yields the lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \lambda m^2\varphi^3 - 2\lambda\varphi(\partial_\mu\varphi)^2 - \frac{1}{2}\lambda^2 m^2\varphi^4 - 2\lambda^2\varphi^2(\partial_\mu\varphi)^2.$$

   There is a φ^3 vertex (with outgoing momenta k_j) with factor $-i(3!\lambda m^2 + 2!2\lambda(ik_1\cdot ik_2 + ik_2\cdot ik_3 + ik_3\cdot ik_1))$. This may be simplified by noting that $0 = (\sum k_j)^2 = \sum k_j^2 + 2\sum_{i<j} k_i\cdot k_j$, so the vertex factor is $-i\lambda(6m^2 + 2(k_1^2 + k_2^2 + k_3^2))$.

   There is also a φ^4 vertex with factor $-i(4!/2\lambda^2 m^2 + 2!2!2\lambda^2\sum ik_i\cdot ik_j)$, which may be simplified similarly to yield $-i\lambda^2(12m^2 + 4(k_1^2 + k_2^2 + k_3^2 + k_4^2))$.

   At tree level, $\varphi\varphi \to \varphi\varphi$ consists of the diagrams



The first diagram is

$$\frac{-i(-i\lambda)^2(6m^2+2(p_1^2+p_2^2+(p_1+p_2)^2))(6m^2+2(p_3^2+p_4^2+(p_1+p_2)^2))}{(p_1+p_2)^2+m^2-i\epsilon}$$
$$= 4i\lambda^2\frac{(m^2+(p_1+p_2)^2)^2}{(p_1+p_2)^2+m^2-i\epsilon} = 4i\lambda^2(m^2-s),$$

where we have used that external particles are on shell, $p_j^2 = -m^2$. The next two diagrams yield $4i\lambda^2(m^2-t)$ and $4i\lambda^2(m^2-u)$ (hence the first three diagrams are known as s, t and u channels respectively). Finally, the last diagram is

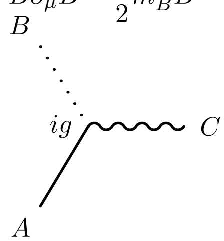
$$-i\lambda^2(4(p_1^2+p_2^2+p_3^2+p_4^2)+12m^2) = 4i\lambda^2 m^2.$$

Hence

$$i\mathcal{M} = 4i\lambda^2(4m^2-s-t-u) = 0.$$

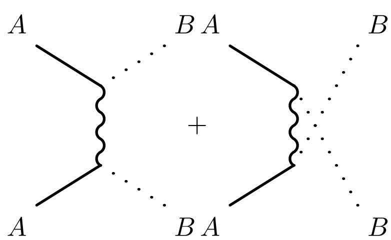
4. For
$$\mathcal{L} = -\frac{1}{2}\partial^\mu A\partial_\mu A - \frac{1}{2}m_A^2A^2 - \frac{1}{2}\partial^\mu B\partial_\mu B - \frac{1}{2}m_B^2B^2 - \frac{1}{2}\partial^\mu C\partial_\mu C - \frac{1}{2}m_C^2C^2 + gABC$$

, we could read out the interaction vertex:



Then, in tree level we have:

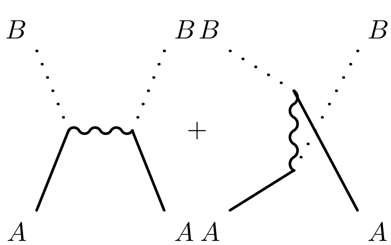
$AA \to BB$



$$\tau = g^2\left(\frac{1}{-t+m_C^2} + \frac{1}{-u+m_C^2}\right)$$

And

$AB \to AB$



$$\tau = g^2\left(\frac{1}{-s+m_C^2} + \frac{1}{-u+m_C^2}\right)$$

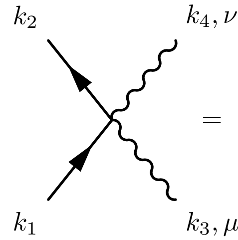
The amplitudes for other processes are 0.

In fact, for this theory, we can make an argument for some of these processes based its symmetry property. We could find a symmetry:

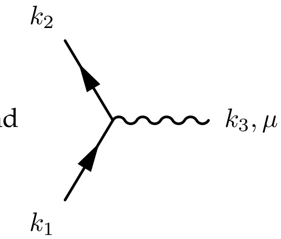
$$A, B \to -A, -B$$
$$A, C \to -A, -C$$
$$B, C \to -B, -C$$

Thus, for example, for process $AA \to AB$, using the symmetry between A and C,we have $\langle AAAB \rangle = \langle (-A)(-A)(-A)B \rangle = 0$ and we know that the amplitude would be 0 not only at tree level, but an exact result. (From Farid Salazar Wong's answer)

5. In scalar QED, we have 2 interaction vertexes:



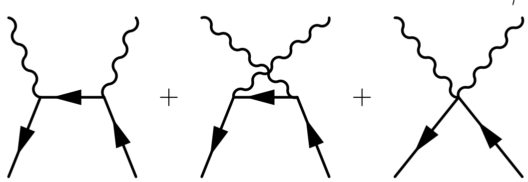
$= -i2e^2 g^{\mu\nu}$ And



$= ie(k_1 + k_2)^\mu$

(a)

$\tilde{e}^+\gamma \to \tilde{e}^+\gamma$

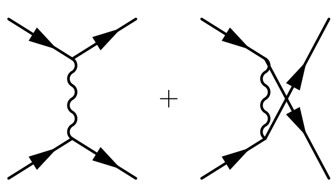


$$\tau = e^2 [\frac{4k^\mu_{e^+in} k^\nu_{e^+out}}{-s+m^2} + \frac{4k^\nu_{e^+in} k^\mu_{e^+out}}{-u+m^2} - 2g^{\mu\nu}] \epsilon^*_{in\mu} \epsilon_{out\nu}$$

where to simplify the expression we have used the fact that each photon polarization is transverse to the momentum, $\epsilon_\mu(k)k^\mu = 0$.

(b)

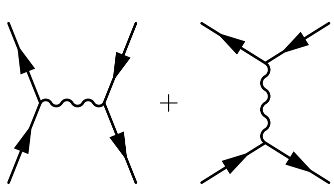
$\tilde{e}^-\tilde{e}^- \to \tilde{e}^-\tilde{e}^-$



$$\tau = e^2 [\frac{(k_{1in} + k_{1out})_\mu (k_{2in} + k_{2out})^\mu}{-t} + \frac{(k_{1in} + k_{2out})_\mu (k_{2in} + k_{1out})^\mu}{-u}]$$

(c)

$\tilde{e}^-\tilde{e}^+ \to \tilde{e}^-\tilde{e}^+$



$$\tau = e^2 [\frac{(k_{1in} - k_{2in})_\mu (k_{1out} - k_{2out})^\mu}{-s} + \frac{(k_{1in} + k_{1out})_\mu (-k_{2in} - k_{2out})^\mu}{-t}]$$