Homework 4

1. **Wick's theorem for finite-dimensional Gaussian integrals**

   Let x_i, $i = 1, \dots N$ be N real variables. We are interested in Gaussian integrals of the form

   $$\int_{-\infty}^{+\infty} dx_1 \cdots \int_{-\infty}^{+\infty} dx_N \; e^{-\frac{1}{2}\sum_{i,j=1}^{N} x_i A_{ij} x_j} \,,$$

   where A is a real symmetric matrix with positive eigenvalues.

   Define the "correlator"

   $$\langle x_{i_1} \cdots x_{i_{2n}} \rangle \equiv \frac{\int dx_1 \cdots dx_N \; e^{-\frac{1}{2}\sum x_i A_{ij} x_j} \; x_{i_1} \cdots x_{i_{2n}}}{\int dx_1 \cdots dx_N \; e^{-\frac{1}{2}\sum x_i A_{ij} x_j}} \,.$$

   "Wick's theorem" says that

   $$\langle x_{i_1} \cdots x_{i_{2n}} \rangle = \sum_{\text{pairings}} \prod_{\text{each pair}} A^{-1}_{i_a i_b} \,,$$

   where A^{-1} is the matrix inverse. Sketch a proof of this theorem. Apply it to compute

   $$I = \frac{\int dxdy \; x^4 y^2 \, e^{-(x^2+xy+2y^2)}}{\int dxdy \, e^{-(x^2+xy+2y^2)}} \,.$$

2. Srednicki problem 9.1

3. Srednicki problem 9.2

4. Srednicki problem 9.3

5. Srednicki problem 9.5. (Perturbation theory from the Dyson expansion. An operator version of Wick's theorem would be the next step – we'll discuss this in class.)