Homework 3

1. Consider the theory of a scalar field in d spacetime dimension, defined by the action

$$S = \int d^d x \, \left(-\frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right) ,$$

where $V(\varphi)$ is an arbitrary potential. We want to study invariance of this theory under *scale transformations* of the spacetime coordinates,

$$x_\mu \to x'_\mu \equiv \lambda x_\mu \, ,$$

where λ is a positive real number. The field is assumed to rescale with a certain weight Δ,

$$\varphi'(x') = \lambda^{-\Delta} \varphi(x) \, .$$

(a) For which choices of Δ and $V(\varphi)$ is the theory scale invariant in dimension d? (The answer will depend on d.)

(b) Find the associated conserved current J_μ. Hint: the infinitesimal transformation is (writing $\lambda = 1 + \epsilon$)

$$\delta\varphi(x) = \varphi'(x) - \varphi(x) = \lambda^{-\Delta}\phi(x/\lambda) - \varphi(x) = -\epsilon(\Delta + x^\mu \partial_\mu)\varphi(x) + O(\epsilon^2) \, .$$

2. Srednicki problem 7.4

3. Srednicki problem 8.3

4. Srednicki problem 8.5

5. Srednicki problem 8.7

6. Find an analytic expression for the position space Feynman propagator $\langle 0|T\phi(x)\phi(0)|0\rangle$ of real scalar field of mass m. How does the expression simplify for $m = 0$? Repeat the problem (only for $m = 0$) for the advanced propagator.

7. Srednicki problem 13.1