## Homework 1

1. Prove that the product of two orthochronous Lorentz transformations is orthochronous. Show that proper orthochronous Lorentz transformations preserve the sign of the time component of time-like and null 4-vectors. Show that, for a space-like vector, an appropriate proper orthochronous Lorentz transformation can change the sign of the time component or set it equal to zero. (Recall that a Lorentz transformation is *orthochronous* if $\Lambda^0{}_0 \geq 1$, and *proper* if $\det \Lambda = 1$.)

2. **Phonons**

   Consider a one-dimensional "crystal", described by the Hamiltonian

   $$H = \frac{1}{2} \sum_{-\infty}^{\infty} \left[ \pi_n^2 + (\varphi_n - \varphi_{n-1})^2 + m^2 \varphi_n^2 \right] .$$

   Here $\varphi_n$ stands for the displacement of the $n$th atom, at position $x = na$ on the lattice (where the lattice spacing $a$ has been set to one) and $\pi_n$ is the canonically conjugate variable. The second term in $H$ describes the coupling between nearest neighbors, while the third term is a restoring potential to the equilibrium position. The usual equal time canonical commutation relations (for the Heisenberg picture operators) apply,

   $$[\varphi_m(t), \pi_n(t)] = i\delta_{mn} .$$

   In analogy with our treatment of the continuum scalar field, let us write the Fourier expansion

   $$\varphi_n = \int_{-\pi}^{\pi} \frac{dk}{(2\pi)2\omega_k} \left( a_k \, e^{-i\omega_k t + ikn} + a_k^\dagger \, e^{i\omega_k t - ikn} \right) .$$

   (a) What is $\omega_k$ as a function of $k$ and $m$?

   (b) Why is the momentum taking values in the interval $[-\pi, \pi]$?

   (c) Derive the commutation relations for the $a_k$ and $a_k^\dagger$ oscillators.

   (d) Write the Hamiltonian in terms of $a_k$ and $a_k^\dagger$. Interpret your result physically.

   (e) Finally, use dimensional analysis to restore the lattice spacing $a$ in your formulas, and take the continuum limit $a \to 0$.

3. Problem 3.1 in Srednicki.

4. Problem 3.2 in Srednicki.

5. Problem 3.5 in Srednicki.

6. Prove equ.(3.32) in Srednicki.