Gauged Supergravity and Holographic RG Flows

Stonybrook, December, 2001

Based upon:

- hep-th/9904017; hep-th/9906194; hep-th/004063
  Freedman, S. Gubser, K. Pilch and NPW
- Recent work with R. Corrado, M. Gunaydin and M. Zagermann
  to appear

Overview

- Teaching an old dog new tricks: The AdS/CFT Correspondence
- Holgraphic Description of RG flows
- Open problems in holographic field theory and supergravity
  - Half-maximal supersymmetric models

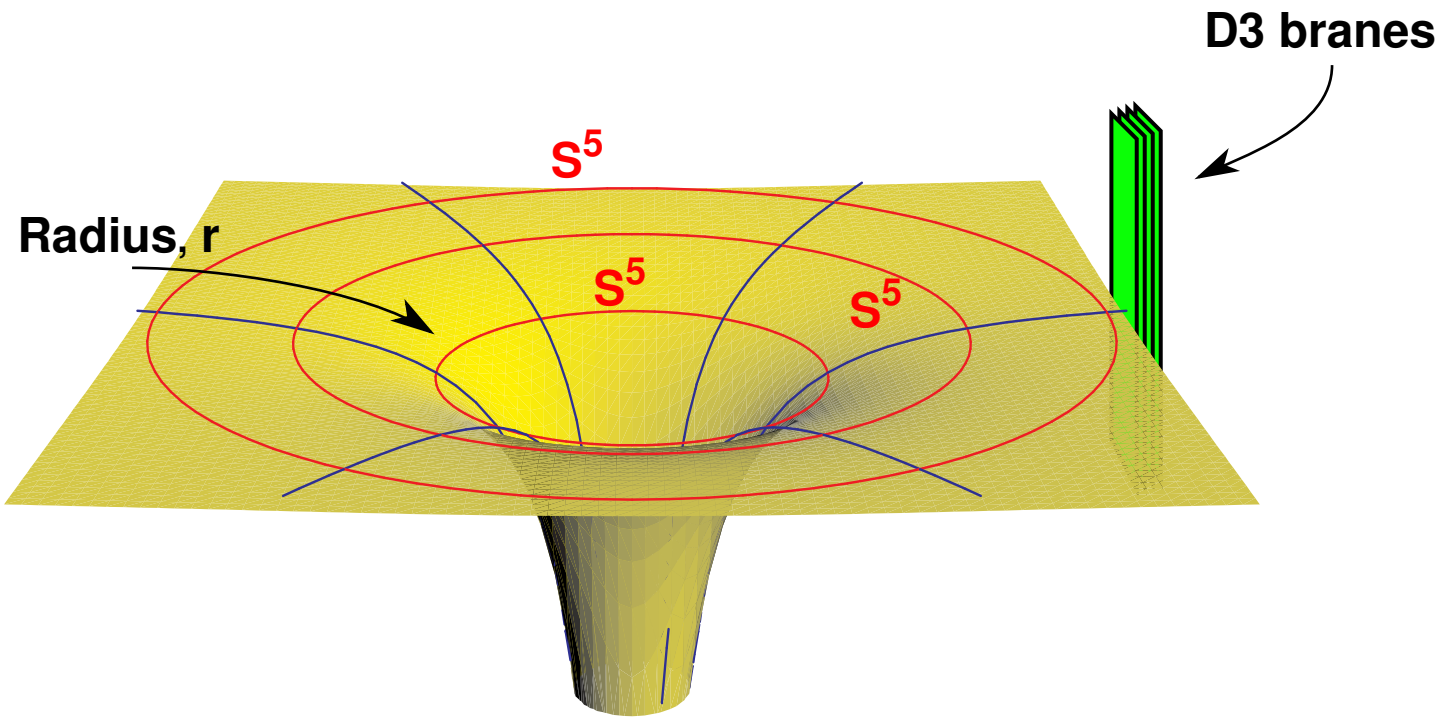
    $\mathcal{N}$ = 2 Quiver Gauge theories $\leftrightarrow$ $\mathcal{N}$ = 4 supegravity + tensor multplets
  - Half-maximal supersymmetric flows

    $\mathcal{N}$ = 2 Seiberg Witten flows
  - Supersymmetric AdS geometries
- Other issues

Yang-Mills theory and AdS/CFT

D3-branes in IIB Supergravity

$\mathcal{N} = 4$ Yang-Mills in strongly coupled, Large N limit



D3 branes + r $\leftrightarrow$ AdS_5

Operators on the brane $\leftrightarrow$ Sources in Supergravity

Correlators and states $\leftrightarrow$ Classical supergravity solutions

Five-dimensional perspective:

- View S^5 as an internal space
- Decompose theory into generalized Fourier modes on S^5
- Space-time = D3 branes + radius, r
- Interpret r as an RG scale for theory on brane

Interpreting the metric

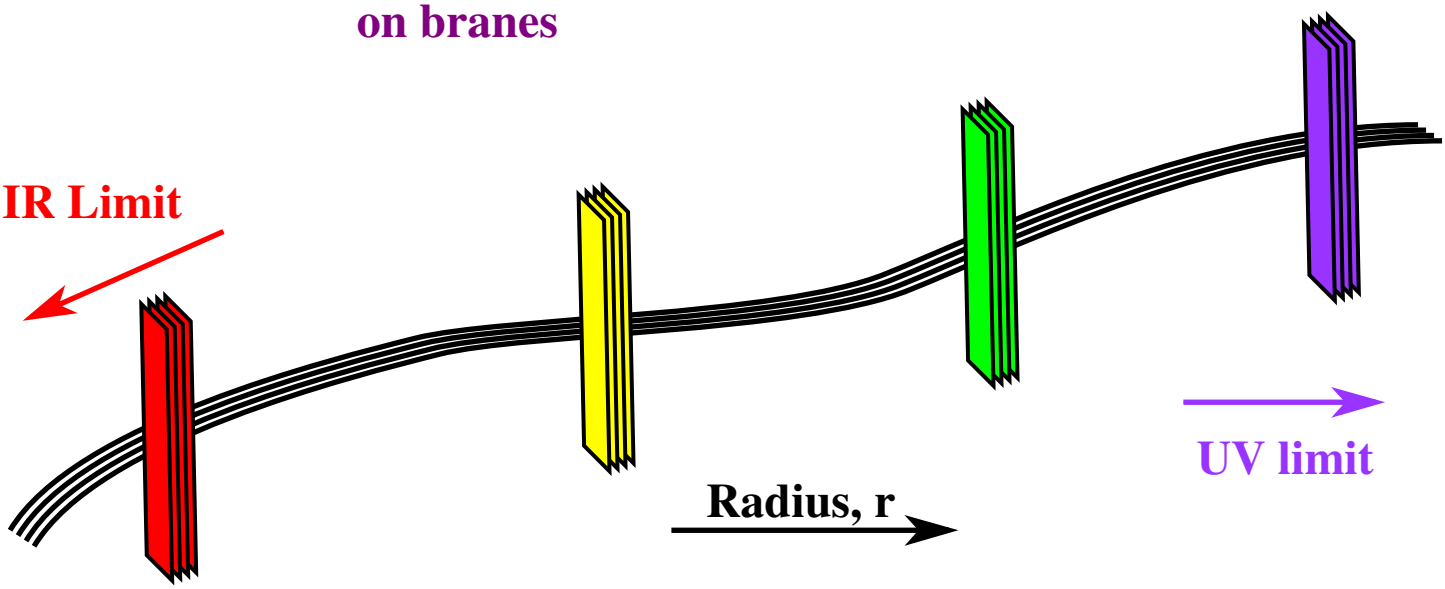
$$ds_{10}^2 = \Omega^2(y)\left(e^{2A(r)} \eta_{\mu\nu}\, dx^\mu\, dx^\nu \;\; dr^{\,2} \right) + ds_5^2(y)$$

warp factor

Physical scale on branes

D3 brane

internal



- **Localize D3-branes at infinity** ⟷ **Conformal, UV limit**
  Localize D3-branes finite r ⟷ Explicit cut-off
- **Red shifts** ⟷ **Wilsonian coarse graining**
- **Cosmological Entropy** ⟷ **c-function / theorem**
- **No hair theorems** ⟷ **Universality in IR limit**
- **Gauged supergravity potentials** ⟷ **Phase diagrams / flows** from relevant perturbations
- **Consistent truncation** ⟷ **Large N structure of OPE in E.M. supermultiplets**

Infra-red Fixed Points

I. Perturb the $\mathcal{N} = 4$ theory: $(A_\mu,\ \lambda^1, \ldots, \lambda^4,\ X^1, \ldots, X^6)$

Freedman, Gubser, Pilch and Warner, hep-th/9901017

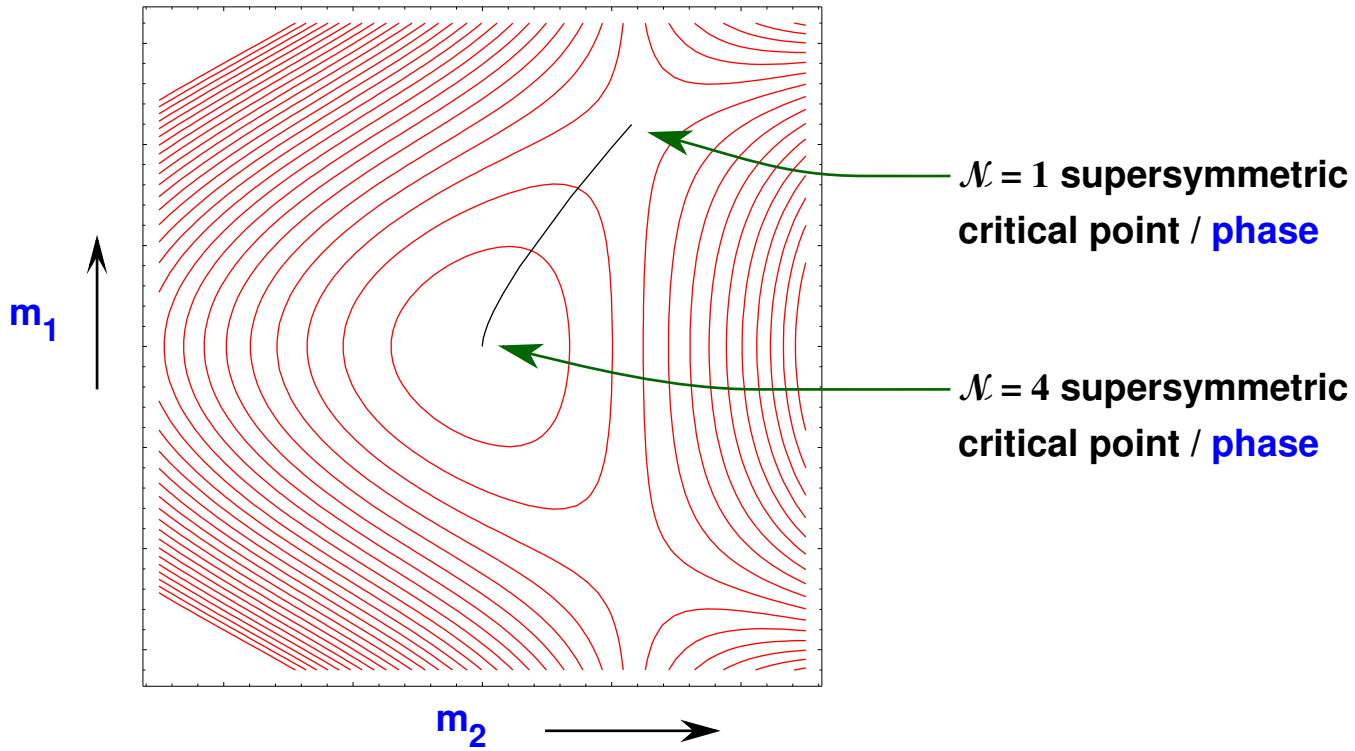
Give mass to an $\mathcal{N} = 1$ chiral multiplet: $W = W_0 + m\ \mathrm{Tr}(\Phi^2$

$\Delta \mathcal{L} = m_1\ \mathrm{Tr}(\lambda^1\lambda^1) + m_2\ \mathrm{Tr}((X^1)^2 + (X^2)^2);$ $\quad m_2 = m_1^{\,2} = m^2$

Non-trivial $\mathcal{N} = 1$ fixed point (Leigh and Strassler) for flow as $m \to \infty$

$$\frac{C_{IR}}{C_{UV}} = \frac{27}{32}$$

Supergravity: steepest descent on a supergravity superpotential, $\mathcal{W}$:



II. Perturb an $\mathcal{N} = 2$ superconformal quiver theory:

Klebanov and Witten hep-th/9807080

Gubser, Nekrasov and Shatashvili hep-th/9811230

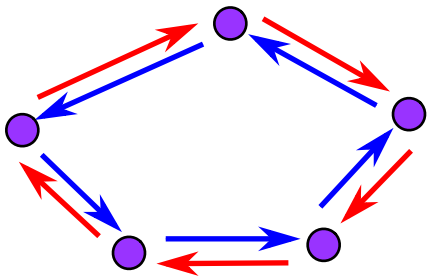
$$\frac{C_{IR}}{C_{UV}} = \frac{27}{32}$$

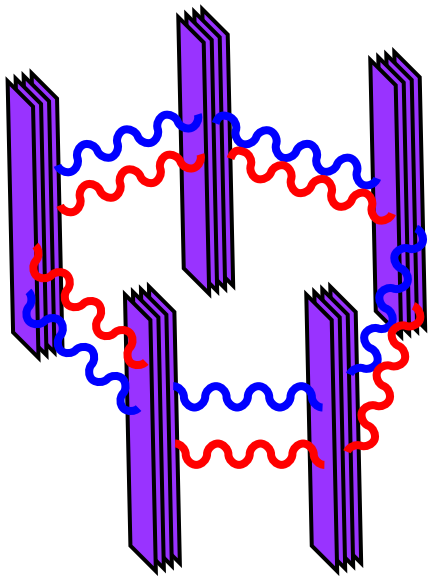
Quiver Theories and Orbifolds

$\mathcal{N} = 2$, $(SU(N))^p$ Yang-Mills

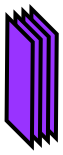
+ bi-fundamental matter

Z_p Orbifold of p x N D3-branes





= SU(N) =



$\rightarrow$ = (N, $\overline{\text{N}}$) =

$\leftarrow$ = ($\overline{\text{N}}$, N) =

Superconformal with R-symmetry = SU(2) x U(1)

Untwisted sector: Vector multiplets

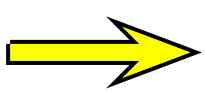
$$\left(A^{(k)}_{\mu};\ \lambda_1^{(k)},\ \lambda_2^{(k)};\ \phi^{(k)} = X_1^{(k)} + i\, X_2^{(k)} \right) \qquad k = 1, \ldots, p$$

Twisted sector: Hypermultiplets $\left(\psi_1^{(k)},\ \psi_2^{(k)};\ A^{(k)},\ B^{(k)} \right)$

Mass terms / flow: $\Sigma_k\ m_{(k)} \left(\Phi^{(k)}\right)^2$

Klebanov and Witten: $m\ \left(\left(\Phi^{(1)}\right)^2 - \left(\Phi^{(2)}\right)^2 \right)$ p=2, $\mathcal{Z}_2$ odd

Freedman, Gubser, Pilch and Warner: $m\ \left(\left(\Phi^{(1)}\right)^2 + \left(\Phi^{(2)}\right)^2 \right)$ $\mathcal{Z}_2$ even



Leigh-Strassler fixed points

Five-dimensional, gauged supergravity and quiver flows

Gauged $\mathcal{N} = 4$ supergravity + Vector or Tensor multiplets:

Dall'Agata, Herrmann and Zagermann, hep-th/0103106

R-symmetry on the brane = Gauge group: SU(2) x U(1) (supergravity)

$m_k^{ab} \lambda_a^{(k)} \lambda_b^{(k)} + \text{c.c.} \longleftrightarrow 3\,(+2) + 3\,(-2)$

$m_k \phi^{(k)\,2} + \text{c.c.} \qquad 1\,(+4) + 1\,(-4)$

$\tau_k = \frac{4\,i}{g_k^{\,2}} \frac{\theta_k}{2} \longleftrightarrow 1\,(0) + 1\,(0)$

Scalar content of a pair of five-dimensional, charged tensor multiplets

Gauged $\mathcal{N} = 4$ supergravity + 2p Tensor multiplets: charges (+2) and (-2)

Scalar Manifold (Supergravity) $\longleftrightarrow$ Couplings in Gauge theory

$$SO(1,1) \times \frac{SO(5, 2p)}{SO(5) \times SO(2p)} \supset SO(3) \times SO(2, 2p)$$

$SU(2)_{\mathcal{R}} \times U(1)_{\mathcal{R}} \times SU(p, 1)$ ← Invariance of supergravity potential

$\frac{SU(p, 1)}{SU(p) \times U(1)}$ = Non-compact CP_p parametrizing couplings: τ_k

Generalizes:

SU(1,1)/U(1) of IIB supergravity $\longleftrightarrow$ $\mathcal{N} = 4$ Yang-Mills Coupling, τ

Unexpected Consequences: SU(p) Symmetry

Untwisted scalar sector in $\mathcal{N} = 8$ supergravity

+

Twisted sector scalars from 2(p-1) additional tensor multiplets

$=$ $\mathcal{N} = 4$ supergravity + two charged tensor mutiplets $\Rightarrow \dfrac{SO(5, 2)}{SO(5) \times SO(2)}$

$=$ (p-1) x (5,2) of SO(5) x SO(2) $\Rightarrow \dfrac{SO(5, 2p)}{SO(5) \times SO(2p)}$

Flow of FGPW: $m\,((\Phi^{(1)})^2 + (\Phi^{(2)})^2)$

lies entirely in scalars of untwisted sector

Flow of KW: $m\,((\Phi^{(1)})^2 - (\Phi^{(2)})^2)$

lies entirely in scalars of twisted sector

U(p) symmetry of $\mathcal{N} = 4$ supergravity potential

SU(p) symmetry $\Rightarrow$ Critical point of FGPW extends to a (p-1)-dimensional critical surface

Field Theory:

Flows driven by $\Sigma_k\, m_{(k)}\, (\Phi^{(k)})^2$ go to a smooth, SU(p) symmetric family of IR fixed points parametrized by $\tau_k{}^2 / m_k$ considered as homogeneous coordinates on CP_{p-1}.

IIB Supergravity: Bizarre

- FGPW flow: topologically trivial flux on S^5/Z_2
- KW flow: topologically non-trivial flux on blow-up of A_1 singularity on S^5/Z_2

U(p)

Comments:

- Family of equivalent fixed points with same central charge: parametrized by initial "velocities", $m_{(k)}$, of flow.

- This is a "large N result:" the U(p) symmetry is broken by anomalies at finite N.

  Residual Symmetry $\quad U(p) \supset \mathcal{G} \supset (Z^{2N})^p$ ($\mathcal{G}$: finite)

- Discrete symmetry at finite N between FGPW and KW flow?

- Singular behavior at finite N when one or more $m_{(k)}$ vanish?

- Does consistent truncation work for $\mathcal{N} = 4$ supergravity?
  - Unknown territory...
  - We have solutions in both five and ten dimensions.
    - Solutions coincide at linearized level
    - Same initial velocities
    - Same supersymmetry
    - Same $\mathcal{R}$-symmetry

- Try to find the SU(2) family of solutions in IIB supergravity that interpolates between a topologically trivial flux on S^5/Z_2 and a topologically non-trivial flux on the blow-up of an A_1 singularity on S^5/Z_2.

  Corrado, Pilch and Warner: work in progress

Another duality that trades 3-form flux for Kähler moduli ....

Open Problems: Supersymmetric *RR*-Geometries

Geometry of supersymmetric solutions *with background RR-fluxes?*

Ricci flat manifolds + $\mathcal{N} = 2$ supersymmetry: Kähler

Ricci flat manifolds + $\mathcal{N} = 4$ supersymmetry: Hyperkähler

What is the analogue of this for holographic RG flow solutions?

The issue for flow solutions in IIB supergravity:

- **The flows generally start from AdS_5, with a background 4-form RR-tensor gauge field**
- **Softly broken $\mathcal{N} = 4$ Yang-Mills: fermion masses are holographically dual to 2-form tensor gauge fields**
- **The dilaton and axion are dual to the Yang-Mills gauge couplings, and thus generically run.**

→ **Interesting supersymmetric flow solutions involve fluxes for all the background tensor gauge fields**

Classification theorems for such solutions: very few, and none of them relevant to the important physical holographic flows.

Important Example: (unsolved)

Find the Holographic Dual of the $\mathcal{N} = 2$ Seiberg-Witten effective action

Holography and the $\mathcal{N} = 2$ Seiberg-Witten effective action

Simplest version:

$\mathcal{N} = 4$ Yang-Mills $\longrightarrow$ $\mathcal{N} = 2$ Yang-Mills + massive hypermultiplet

Coulomb branch: $u_n = \mathrm{Tr}\,(\phi^n)$

$\downarrow$

D3-brane distribution Source: $\rho_y = \Sigma\, u_n\, y^n$

Complex scalar, ϕ $\longrightarrow$ Complex coordinate, y

Problem: Find the general $\mathcal{N} = 4$ supersymmetric flow solution determined by an arbitrary source function, ρ_y , of two variables

What is known: the solution for one point in the moduli space

$$\rho_y = a^2\, |y|^{2\ 1/2}$$

A uniform disk-like distribution of D3-branes

Pilch and Warner; hep-th/0006066
Buchel, Peet and Polchinski; hep-th/0008076
Evans, Johnson and Petrini; hep-th/0008081

The general $\mathcal{N} = 8$ supersymmetric flow solution (the Coulomb branch of $\mathcal{N} = 4$ Yang-Mills) is well-known, and is determined by an arbitrary harmonic source function, H_y , of six variables. The half-maximal supersymmetric solutions shouldn't be much more difficult....

Harder problem: Find the general $\mathcal{N} = 4$ supersymmetric flow solution for the quiver gauge theories

Other Issues

- $\mathcal{N} = 1$ supersymmetric flows: generalized Kähler structure in the presence of *RR* fluxes?
  - Brane probe results find the Kähler structure on the moduli space of the probe.

    Johnson, Lovis, C. Page, hep-th/0107261

- Non-compact gaugings: Holographic interpretation???
  - Vacua: De-Sitter space → All supersymmetry broken
  - Domain wall solutions can be (half-maximally) supersymmetric

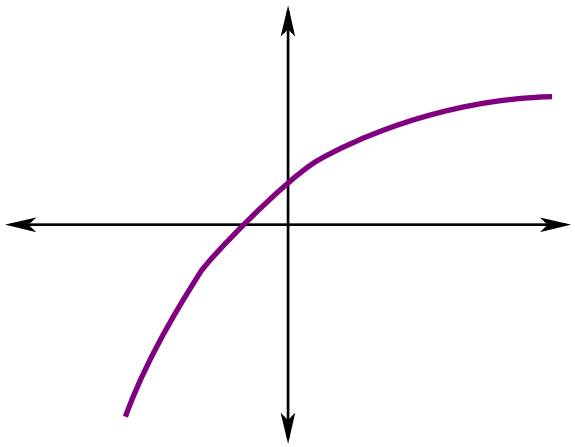
    Hull, hep-th/0110048; Gibbons and Hull hep-th/0111072

- Potentials without critical points:

  Domain wall solutions...

  ⇓ ???

  Gauged Supergravity description + Generalizations and variations on Duality Cascade of Klebanov and Strassler? hep-th/0007191



- Most of the exactly known flow solutions have been obtained by lifting solutions of gauged supergravity
  - Systematics of lifting
  - General methods of construction in 10 or 11 dimensions

Conclusions

- **Gauged supergravity is a very valuable tool in the study of holographic RG flows**

- **$\mathcal{N} = 4$ gauged supegravity coupled to tensor multiplets can be used to study a class of $\mathcal{N} = 1$ supersymmetric flows in large N, $\mathcal{N} = 2$ quiver gauge theories**

- **The large N, $\mathcal{Z}_p$ quiver models have a (p-1)-dimensional surface, CP_{p-1}, of $\mathcal{N} = 1$ supersymmetric fixed points, and all these fixed point theories are equivalent under the action of an SU(p).**

- **In ten dimensions, this SU(p) must act as a "generalized duality symmetry", mapping compactifications with fluxes on $S^5/\mathcal{Z}_p$ to compactifications with blow-ups of the A_{p-1} singularity:**

Flux $\longleftrightarrow$ **Kähler Moduli**

- **Important open question in holographic RG flows:**

Geometry of supersymmetric compactifications with *RR* fluxes???

  - **Holographic description of Seiberg-Witten actions**
  - **Holographic interpretation of the many supersymmetric domain-wall solutions?**