Conformal Supergravity and String Theory

A. Tseytlin

String theory → supersymmetry → super gravity

$\alpha' \to 0$

2-d (Conformal) Supergravity → string theory

2-d

Deser, Zumino
Brink, di Vecchia, Howe
Brink, Schwarz
Polyakov
Fradkin, A.T.

4-d Quantum Gravity

String theory
(74, 82-84)

[extra massive states, UV cutoff]

"R^2" gravity / conformal supergrv.
(77-84)

[extra massive ghosts]

4-d: Kaku, Townsend, van Nieuwenhuizen
Bergshoeff, de Roo, de Wit ...
Fradkin, A.T.

Relations between string theory and conformal supergravity (98)

Ferrara et al
Liu, A.T.

AdS / CFT

gauged sugra in AdS_5 → conformal sugra in $R^{1,3}$, induced by SYM, etc.

Generalization: conformal higher spins in $R^{1,3}$ ↔ massless higher spins in AdS_5

$\lambda \to 0$:

Witten
Sundborg
Vasiliev
Sezgin, Sundell

2-d Conformal sugra (coupled to matter):

NSR string

(1,1) : $(e_\mu^a, \chi_\mu) + (x^m, \psi^m)$ $\quad m = 1, \dots, D$

$$\mathcal{L} = \sqrt{g}\Big(g^{\mu\nu}\partial_\mu x^m \partial_\nu x^m + i\bar{\psi}^m \not{\partial}\psi^m + (\partial_\mu x^m + \bar{\chi}_\mu \psi^m)\bar{\psi}^m \gamma^\rho \gamma^\mu \chi_\rho \Big)$$

diff. gauge : $g_{\mu\nu} = e^{2\rho}\delta_{\mu\nu}$, $\chi_\mu = \partial_\mu x$

Quantum theory: Polyakov 81

$$Z \sim \int \mathcal{D}x\, \mathcal{D}\psi\, e^{-I}$$

Critical string: demand superconformal symmetry at quantum level → no anomaly, no (ρ, x) dependence induced

$$\ln Z \sim (D-10)\int d^2z \, (\partial\rho\,\bar{\partial}\rho + i\bar{\psi}\partial\psi)$$

↦ $D = 10$

2-d conformal sugra coupled to 10 supermultiplets is consistent at quantum level with same # of d.o.f. as at classical level

$n = 2$ string: ("charged string") Ademollo et al, Brink Schwarz

extended conformal supergravity

$(e_\mu^a, \underline{\chi_\mu^i}, \underline{A_\mu}) + (\hat{x}^m, \hat{\psi}^m)$ $\quad \hat{x}^m = \underline{x}^m + i y^m$ $\quad (m = 1, \dots, D)$

$$\mathcal{L} = \sqrt{g}\Big[\, \partial\hat{x}^m\, \partial\hat{x}^{*m} + i\,\bar{\psi}^m \gamma^r \mathcal{D}_r(A)\,\psi^m + (\partial_\mu x^m + \bar{\chi}_\mu \psi^m)\,\bar{\psi}^m \gamma\rho\gamma^r \chi_\rho + h.c.\Big]$$

$$\mathcal{D}\psi = \Big(\partial_\mu + \tfrac{1}{4}\omega_\mu + iA_\mu\Big)\psi + \bar{\chi}\partial\chi\psi$$

$$\chi = \chi^1 + i\chi^2$$

$$\delta e^a_\mu = \underline{\Omega}\, e^a_\mu + \ldots$$

$$\delta\chi_\mu = i(\xi + \gamma\underline{\theta})\chi_\mu + \gamma_\mu \Lambda + \ldots$$

$$\delta A_\mu = \partial_\mu \xi + \varepsilon_{\mu\rho}\partial^\rho \underline{\theta} + \ldots$$

Classical gauge: $e^a_\mu = \delta^a_\mu$, $\chi_\mu = 0$, $A_\mu = 0$

Weyl + $U(1)$ anomaly:

$$e^a_\mu = e^{\underline{\rho}}\delta^a_\mu, \quad \chi_\mu = \partial_\mu \underline{\chi}, \quad A_\mu = \varepsilon_{\mu\nu}\partial^\nu \underline{\sigma}$$

(ρ, σ, λ) - multiplet decouples for $D = 2$

Fradkin, T. 81

$$\ln Z \sim (D-2)\int d^2x \Big[(\partial\rho)^2 - (\partial\sigma)^2 + i\,\bar{\chi}\,\partial\!\!\!/\,\chi\Big]$$

Simplification of extended conformal supergravity:

find anomaly in $U(1)$ (i.e. A_μ) sector

$$\int (\partial\rho)^2 \sim \int R\,\Box^{-1} R$$

$$\int (\partial\sigma)^2 \sim \int F_{\mu\nu}\,\Box^{-1} F_{\mu\nu} \sim \int (A^\perp_\mu)^2$$

induced (Schwinger) action: $\log\det \mathcal{D}(A)$

Coefficient of $(A_\mu^\perp)^2 \sim (\partial\sigma)^2$ is easy to compute

pure conf. sugra : -2 matter : D

anomaly cancellation not possible for $n>2$ sugra

$n=4$ P.v.N. Seesin

Conformal supergravity in 4 dimensions

Many analogies with 2-d case

N-extended conf. sugra + K vector multiplets (SYM: dim $G = K$)

Cancellation of anomalies – consistent (finite!) theory

for $N=4$, $K=4$ only

Fradkin, A.T. 82-83

Absence of conformal and SU(4) anomalies necessary for quantum consistency (otherwise non-renormalizable at 2 loops)

1-loop ∞ or conf. anomalies only: $N=2$ non-renormaliz. theorem Howe, Stelle Townsend 84

$T^\mu_\mu \sim b_4$

$\Gamma_\infty = \ln \Lambda \int d^4x \; b_4$ Duff

$$\boxed{b_4 = \beta_1 R^* R^* + \beta_2 \left(W + \tfrac{1}{3}\nabla^2 R\right)} \qquad W = R^2_{\mu\nu} - \tfrac{1}{3}R^2 = \tfrac{1}{2}\left(C^2_{\ldots\ldots} - R^* R^*\right)$$

$N=4$ vector multiplet (A_μ, Ψ^i, X^{ij})

$1 \quad 4 \quad 6$

$\beta_1 = 0, \quad \beta_2 = \frac{1}{2}$ (exact to all loops)

$g_{\mu\nu} = e^{2\rho}\tilde{g}_{\mu\nu}$: induced action $= \Gamma_{anom}[\rho,\tilde{g}] + \Gamma^{[\tilde{g}]}_{non\text{-}loc.}$

Fradkin, A.T.
Riegert, 84

$$\Gamma_{anom} \sim \int d^4x\sqrt{\tilde{g}}\Big[(\beta_1 \tilde{R}\tilde{R}^* + \beta_2 \tilde{W})\rho$$
$$+ (\beta_2 - 2\beta_1)\Big((\partial\rho\partial\rho)^2 + 2(\partial\rho)^2\tilde{\Delta}\rho$$
$$+ 2(\tilde{R}_{\mu\nu} - \tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R})\partial^\mu\rho\partial^\nu\rho\Big)$$
$$- \beta_2\Big(\tilde{\Delta}\rho + \partial\rho\partial\rho - \tfrac{1}{6}\tilde{R}\Big)^2\Big] \sim \int \beta_2 \rho D^4\rho + \dots$$

$N=4$ SYM coupled to conformal supergravity

Bergshoeff, de Roo
de Wit
de Roo, Wagemans

$$(A_\mu, \psi^i, X^{ij}) + (e^a_\mu, V^i_{\mu j}, \varphi, T^{ij}_{\mu\nu}, E_{ij}, \mathcal{D}^{ij}_{kl}; \psi^i_\mu, \Lambda_i, \chi^k_{ij})$$

$i,j = 1,2,3,4$
$SU(4)$

$$\Psi = C + ie^{-\phi}$$

$$\mathcal{L}_{SYM} \sim e^{-\phi} F_{\mu\nu}F^{\mu\nu} + C F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}^i \gamma^\mu \mathcal{D}_\mu \psi_i$$
$$+ X_{ij}\left(-\Delta + \tfrac{1}{6}R\right)X^{ij} - X_{ij}F^{+\mu\nu}T^{ij}_{\mu\nu}$$
$$+ \mathcal{D}^{kl}_{ij}\left(X^{ij}X_{kl} - \tfrac{1}{6}\delta^i_k\delta^j_l|X|^2\right) + \dots$$

$$F^+_{\mu\nu} = F_{\mu\nu} + \tilde{F}_{\mu\nu}$$

linear level = "current" × "field"

$$\mathcal{L}_{CSG} \sim C_{\mu\nu\lambda\rho} C^{\mu\nu\lambda\rho} + \left(F^{i}_{j\mu\nu}(V)\right)^2$$

$$+ \partial^2\varphi^* \partial^2\varphi - 2\left(R^{\mu\nu} - \tfrac{1}{3} g^{\mu\nu} R\right) \partial_\mu \varphi^* \partial_\nu \varphi$$

$$+ \partial^\mu T^{ij+}_{\mu\nu} \partial_\lambda T^{\lambda\nu-}_{ij} - \tfrac{1}{2} R^{\mu}_{\nu} T^{ij+}_{\mu\lambda} T^{\lambda\nu-}_{ij}$$

$$+ E^{ij}\left(-\partial^2 + \tfrac{1}{6} R\right) E_{ij} + \partial^{ij}_{\mu\nu} \partial^{\mu\nu}_{ij}$$

$$+ \bar{\psi}^{i}_{\mu} \partial^3_{\mu\nu} \psi_{\nu i} + \bar{\lambda}^{i} \partial^3 \lambda_i + \bar{\chi}^{\mu}_{ij} \partial\!\!\!/ \chi_{\mu}^{ij} + \dots$$

Contributions to conformal anomaly:

	$h_{\mu\nu}$	ψ_μ	V_μ	$T_{\mu\nu}$	χ	λ	E	φ
d.o.f.	6	−8	2	6	−2	−6	2	4
β_1	$\frac{137}{60}$	$-\frac{173}{180}$	$-\frac{13}{180}$	$\frac{11}{30}$	$\frac{7}{720}$	$\frac{21}{720}$	$\frac{1}{30}$	$\frac{1}{45}$
β_2	$\frac{199}{15}$	$-\frac{149}{30}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$-\frac{1}{60}$	$\frac{1}{30}$	$-\frac{4}{15}$

Field content and anomalies of N-extended CSG's:

$$T^{\mu}_{\mu} \sim \beta_1 R^* R^* + \beta_2 \left(\tfrac{1}{2} C^2_{\lambda\mu\nu\rho} + \left(F^{i}_{j\mu\nu}\right)^2 + \dots\right)$$

$U(N)$, $N = 1, 2, 3$; $SU(4)$

N	$h_{\mu\nu}$	ψ^i_μ	V^i_j	$T^{ij}_{\mu\nu}$	χ^κ_{ij}	λ_i	E_{ij}	φ	β_1	β_2
1	1	1	1	0	0	0	0	0	$\frac{5}{4}$	$\frac{17}{2}$
2	1	2	4	1	2	0	0	0	$\frac{11}{24}$	$\frac{13}{3}$
3	1	3	9	3	9	1	3	0	0	1
4	1	4	15	6	20	4	10	1	0	−2

$N=4$ CSG + SYM ($\dim G = K$) :

$$\beta_1 = 0 + 0 = 0 , \qquad \beta_2 = \frac{1}{2}\cdot K - 2$$

(analogy with 2-d case!)

$\boxed{K=4}$

- same result for coeffs of $F_{\mu\nu}^2(V)$ (simpler!)
- same result from cancellation of $SU(4)$ chiral anomaly

Römer van Nieuwenhuizen 85

$$\partial_\mu J^\mu_5 \sim F_{\mu\nu}(V)\, F^*_{\mu\nu}(V)$$

Consistent with conformal and axial anomaly being in one multiplet

Correspondence with Einstein theory at low energies?

Fujii,... Kaku Townsend 78

Break conformal invariance spontaneously :

one vector multiplet (ghost-like)

$$\mathcal{L} = X\left(-\partial^2 + \frac{R}{6}\right)X + \dots , \qquad \langle X \rangle = a$$

$$\mathcal{L} = \frac{a^2}{6} R + \dots$$

(no Λ-term if flat directions in potential)

Natural in finite superconformal theory with scalars

Fradkin, T. Stromінger Nair 84

Equivalent to making non-anomalous Weyl transformation

$$|X_{ij}|' = \text{const} = a ,$$

$$g'_{\mu\nu} = a^2 |X_{ij}|^2 g_{\mu\nu} \dots$$

(in phase with broken symm.)

Issue of ghosts / unitarity

Euclidean path integral is well-defined (without ghost multiplets)

Conformal supergravity as "induced" theory : AdS / CFT context

$$Z[G] = e^{-\Gamma[G]} = \int d\mathcal{A}\; e^{-S[\mathcal{A},G]}$$

$\mathcal{A} = (A_\mu, \psi^i, X^{ij})$ SYM $SU(N)$

$G = (g_{\mu\nu}, \phi, C, \psi^i_\mu, \dots)$ conformal supra background

$$S_{int} = \int d^4x \left[T_{\mu\nu}(\mathcal{A})\, h_{\mu\nu} + F^2_{\mu\nu}(A)\,\phi + \dots \right]$$

Z not superconf. inv. : anomaly

$$\langle T^\mu_\mu \rangle = \frac{2g_{\mu\nu}}{\sqrt{g}} \frac{\delta\Gamma}{\delta g_{\mu\nu}} = -\beta\, \mathcal{L}_{CSG}$$

$$\mathcal{L}_{CSG} = C^2_{\lambda\mu\nu\rho} - R R^* + 2\left(F^i_{\mu\nu j}(V)\right)^2 + 4\,\phi\,\Delta_4\,\phi + \dots$$

$$\partial_\mu \frac{\delta\Gamma}{\delta V_\mu} = \beta\, F_{\mu\nu}(V)\, F^*(V)^{\mu\nu}$$

$$\beta = \frac{N^2}{4(4\pi)^2}$$

$$\Gamma = \Gamma_\infty + \Gamma_{fin}\,, \qquad \Gamma_{fin} = \Gamma_{anom} + \Gamma_{inv}$$

$$\Gamma_\infty = -\beta \ln \Lambda \int d^4x \sqrt{g}\; \mathcal{L}_{CSG}$$

$g_{\mu\nu} = e^{2\rho}\tilde{g}_{\mu\nu}$

$$\Gamma_{anom} \sim \beta \int \left[\left(\tilde{C}^2 - \tilde{R}\tilde{R}^* + \dots \right) \rho + \dots \right.$$
$$\left. + F^{\mu\nu}(V)\, F^*_{\mu\nu}(V)\, \Box^{-2}\, \Box^2 V_\lambda + \dots \right]$$

Quadratic approximation:

$$\Gamma_2 = \beta \int \Big[C^{\mu\nu\lambda\rho} \ln\left(\frac{-\partial^2}{\Lambda^2}\right) C_{\mu\nu\lambda\rho} + 2 F^{\mu\nu} \ln\left(-\frac{\partial^2}{\Lambda^2}\right) F_{\mu\nu} + 4 \partial^2\phi \ln\left(-\frac{\partial^2}{\Lambda^2}\right) \partial^2\phi + \dots \Big]$$

$$\frac{\delta^n \Gamma}{\delta h_{\mu\nu}^{(x_1)} \dots \delta h_{\alpha\beta}^{(x_n)}} \sim \langle T_{\mu\nu}^{(x_1)} \dots T_{\alpha\beta}^{(x_n)} \rangle$$

conformal inv. (at separated points)

AdS/CFT

$N \to \infty$, $\lambda = g_{YM}^2 N \gg 1$: $D=5$, $N=8$ gauged sugra

$$\Gamma[G] = I_{GSG}[\hat{G}(G)] \qquad \hat{G}\big|_{\partial AdS} = G$$

↖ solution of Dirichlet problem

Quadratic terms:

$$\int d^5x \sqrt{g} (\partial\phi)^2 \to \int d^4x \, \phi \, \partial^4 \ln -\frac{\partial^2}{\Lambda^2} \phi$$ Gubser Klebanov Polyakov

$$\int d^5x \sqrt{g} F_{mn}^2 \to \int d^4x \, F^{\mu\nu} \ln -\frac{\partial^2}{\Lambda^2} F_{\mu\nu}$$ Witten

$$\int d^5x \sqrt{g}(\hat{R} - 2\lambda) \to \int d^4x \, \partial^2 \bar{h}^{\perp}_{\mu\nu} \ln -\frac{\partial^2}{\Lambda^2} \partial^2 \bar{h}^{\perp}_{\mu\nu} + \dots$$ Liu, A.T.

$$-2 \int C^{\mu\nu\lambda\rho} \ln -\frac{\partial^2}{\Lambda^2} C_{\mu\nu\lambda\rho} + \dots$$

AdS_{d+1}: $$ds^2 = \frac{R^2}{z^2}(dz^2 + d\vec{x}^2)$$

$$S \sim \int d^{d+1}x \sqrt{g} \left(\partial\phi\partial\phi + m^2\phi^2\right), \qquad \phi\big|_{\partial M} = \phi_0 \quad (z \to 0)$$

$$\phi(z,x) = c\int d^d x' \; \frac{z^{\Delta_+}}{(z^2+|x-x'|^2)^{\Delta_+}}\,\phi_0(x')$$

Witten
Freedman et al

$$\Delta_+ = \tfrac{1}{2}\left(d \pm \sqrt{d^2+4\mu^2}\right), \qquad \mu \equiv mR$$

$$S \sim \int d^d x\, d^d x' \; \frac{\phi_0(x)\,\phi_0(x')}{(\varepsilon^2+|x-x'|^2)^{\Delta_+}}$$

$\varepsilon \sim \Lambda^{-1}$
$(z=\varepsilon \to 0)$

Graviton: $\hat{h}_{zz}\big|_{z=0} = \hat{h}_{z\mu}\big|_{z=0} = 0$, $\hat{h}_{\mu\nu}\big|_{z=0} \sim h_{\mu\nu}(x)$

$$S \sim \int d^d x\, d^d x' \; \frac{h_{\mu\nu}(x')\, H_{\mu\nu\lambda\rho}(x-x')\, h_{\lambda\rho}(x')}{(\varepsilon^2+|x-x'|^2)^{d}}$$

$$H_{\mu\nu\lambda\rho} \sim (\delta_{\mu(\lambda}\delta_{\nu)\rho} - \text{trace}) - \frac{1}{x^2}(\delta_{\nu\lambda}x_\mu x_\rho + 3\text{ terms}) + \frac{4}{x^4}x_\mu x_\nu x_\lambda x_\rho$$

(same as in $\langle T_{\mu\nu}(x)\, T_{\lambda\rho}(x')\rangle$ in a CFT)

Similar relations for higher-order invariant and anomalous terms in Γ and $d=5$ sugra

(CS in $d=5$: $\int V\wedge dV\wedge dV \to \int FF^* \frac{1}{\partial^2}\partial^m V_m$)

Generalizations ?

Finite cutoff ($z=\varepsilon\neq 0$): dynamical gravity * on the brane at $z=\varepsilon$ → conformal sugra fields should be integrated over? (true "induced CSG")

(cf. Randall-Sundrum)

Balasubramanian, Gimon, Minic, Rahmfeld

(* Sugra fields (non-normalizable modes) have ∞ action if bulk is infinite but will have to fluctuate in finite bulk)

How to justify from string theory perspective ?

Higher-spin generalization

Free conformal "pure spins" in flat space ($d=4$)

$$\mathcal{L} = \sum_s \left(\varphi_s \, \partial^{2s} \, P_s \, \varphi_s + \psi_{s-\frac{1}{2}} \, \partial^{2s-1} \not{\partial} \, P_{s-\frac{1}{2}} \psi_{s-\frac{1}{2}} \right)$$

(Weyl-inv operators)

Fradkin A.T. 84

$s = 2, 3/2, 1$: conformal supergravity $\{ \varphi_{\mu_1 \dots \mu_s}, \psi_{\mu_1 \dots \mu_{s-\frac{1}{2}}} \}$

P_s : transverse, traceless; totally symmetric $P_s^2 = P_s$

Dimensionless coupling ; interaction not prohibited in $R^{1,3}$

Fradkin Linetsky 91

higher derivative - non-unitary ; superalgebras & cubic interactions

Relation to higher massless spins in AdS_5

in same way as $CSG_4 \leftarrow GSG_5$

"Induced theory" interpretation:

Free $N=4$ vector multiplet (or $\lambda = 0$ SYM):

conserved traceless higher spin currents

Burgess et al
Anselmi
Konshtein Vasiliev
Fainin

$$J_s \sim X \, P_s \, \partial^s \, X + \dots$$

↖ scalar of vector multiplet

Couple to φ_s:

$$\mathcal{L} = (X \, \partial^2 X + \dots) + \varphi_s(x) \, X \, P_s \, \partial^s \, X + \dots$$

Integrate out vector multiplet fields →

induced action for $\varphi_s, \psi_s, \dots$ (cf. conformal supra case)

$$\ln \det\left(\partial^2 + \varphi_s(x)\, P_s\, \partial^s\right)$$

$$\to \int \varphi_s \left(\frac{\partial^s}{\partial^2} P_s\right)^2 \varphi_s + \ldots \sim \int \varphi_s\, \partial^{2s} \ln\left(-\frac{\partial^2}{\Lambda^2}\right) P_s\, \varphi_s + \ldots$$

$$\boxed{\Gamma_\infty \sim \ln\Lambda \int \varphi_s\, \partial^{2s}\, P_s\, \varphi_s}$$

$$\Gamma_2 \sim \int d^4x\, d^4x'\; \varphi_s(x)\, \frac{\hat{P}_s(x-x')}{(\epsilon^2 + |x-x'|^2)^{s+2}}\, \varphi_s(x')$$

Should follow from solution of Dirichlet problem for higher-spin massless field in AdS_5 : $\int (\hat{\varphi}_s\, \partial^2 \varphi_s + \ldots)$ *

[agreement guaranteed by symmetry considerations]

$\lambda \to 0$ limit of AdS/CFT Witten Sundborg 00'

Quantum 4-d vector multiplet theory : way to reconstruct interactions of φ_s ? (CSG case-prototype)

non-anomalous part of Γ depends only on original pure spin d.o.f.

Why there are such states in spectrum of superstring in $AdS_5 \times S^5$ in $\lambda \to 0$?

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* Light cone gauge description $\varphi_s \to \{\varphi_{s'}\}$ Metsaev 00'

$$\int d^{d+1}x\; \varphi_{s'}\left(\partial^2 - \frac{1}{z^2} M_{s'}\right) \varphi_{s'}, \qquad M_{s'} = \left(s' + \frac{d-4}{2}\right)^2 - \frac{1}{4}$$

"Zero Tension" limit of superstring in $AdS_5 \times S^5$

dimensionless parameters: $\boxed{\sqrt{\lambda} = \frac{R^2}{\alpha'}}$, $\alpha' p_i^2$, g_s

$\lambda \to 0$: non-trivial limit (cf. flat space)

($R \to 0$ or $\alpha' \to \infty$)

Special property of AdS_{d+1} space (cf. sphere):

$$\mathcal{L} = \partial X^M \partial X^M + \Lambda \left(X^M X_M - \sqrt{\lambda} \right)$$

$$X_M X^M = X_{d+1}^2 + X_\mu X_\mu - X_{d+2}^2$$

Sphere: $X^M X_M \geq 0$: $\lambda \to 0 \to X_M = 0$

AdS: $\lambda \to 0$: $X^M X_M = 0$

"string moving on cone in d+2 dim"

Reduction by 2 dimensions: $\tau = \varphi \pm \psi$, $r = e^{\varphi}$, $v = e^{\psi}$

$X_{d+1} \pm X_{d+2} = u, v$, solve for u; $r^2 \equiv X_i X_i$

$$\mathcal{L} = r^2 \left(\partial\tau \partial\tau + (\partial \Omega_{d-1})^2 \right) : R \times S^{d-1}$$

Same in Poincare coordinates: $\tilde{y} = \lambda^{1/4} y$

$$\mathcal{L} = \sqrt{\lambda} \left(\frac{\partial y \partial y}{y^2} + y^2 \partial x_\mu \partial x_\mu \right) \to \boxed{\tilde{y}^2 \partial x_\mu \partial x_\mu}$$

$T \to 0$ limit in flat space

$\alpha' \to \infty$: $\mathcal{L} \sim \frac{1}{\alpha'} \partial x \partial x + \int e^{ipx} \dots$

Schild

Lindstrom
Karlhede

$x \to \sqrt{\alpha'}\, x$, $p \to \frac{1}{\sqrt{\alpha'}}\, p$

momenta $\to \infty$ for $\alpha' \to \infty$

UV limit : space – time conformal invariance ?

$\mathcal{H} \sim \alpha' p^2 + \frac{1}{\alpha'} x'^2 \to \alpha' p^2$ ($\frac{1}{\alpha'} x'^2 \to 0$) decoupled particles

$\mathcal{L} = T \sqrt{\det \partial x \partial x} \to \chi \det \partial x \partial x + T^2 \chi^{-1}$

$\to \chi \det \partial x \partial x$ or $\chi \cdot \left(\sqrt{g}\, \partial x \partial x\right)^2$

Lindstrom et al

χ – auxiliary field

Quantum theory ? Only massless states ?

Curved space / AdS case is different:

$\frac{R^2}{\alpha'} \to 0$ but momenta are fixed

$\chi (\partial x \partial x)^2 \to y^2 \partial x \partial x$

y → radial direction of AdS

Bosonic part of $AdS_5 \times S^5$ action:

S^5 and kinetic term of y of AdS_5 decouple

$$\mathcal{L} = y^2 \partial x^\mu \partial x^\mu$$ (Naive limit!)

y, x^μ independent : $\int \mathcal{D}y \mathcal{D}x \; e^{-I[y,x]}$

Suggests Weyl invariance at boundary

$$\mathcal{L} = y^2 g_{\mu\nu}(x) \partial x^\mu \partial x^\nu$$

$g_{\mu\nu} \to K^2(x) g_{\mu\nu}$, $y \to K^{-1}(x) y$.

(only conformal symmetry if $\frac{\partial y \partial y}{y^2}$ is present)

→ Weyl invariance of eqs of states → conformal higher spins at the boundary ?!

Naive limit: conformal invariance ?
central charge ?
take limit in correlation functions?

Including fermions:

Metsaev, A.T. 98

Covariant κ-symmetry gauge : $\eta^i = 0$

PSU(2,2|4) : $(Q_i, S_i) \to (\theta_i, \eta_i)$ $i = 1,2,3,4$ SU(4)

$$ds^2 = y^2 dx_p dx_p + \frac{dy^p dy^p}{y^2} \qquad (p = 1,2,\dots,6) \quad \mu = 0,1,2,3$$

γ-matrices: (σ^μ, ρ^p) 4+6 split

$$\mathcal{L} = \frac{R^2}{\alpha'}\Big[\, y^2 \left(\partial_a x^\mu - i\theta_i \sigma^\mu \partial_a \bar\theta^i + h.c.\right)^2 + y^{-2}\,\partial y^p \partial y^p - i\,\epsilon^{ab}\partial_a y^p\, \bar\theta^i \rho^p_{ij} \theta^j + h.c.\Big]$$

($\partial y^p \partial y^p \to 0$, $\epsilon^{ab}\partial_a y^p \to 0$)

$$y \to \left(\frac{R^2}{\alpha'}\right)^{-1/2} y\,, \qquad \frac{R^2}{\alpha'} \to 0 \quad :$$

$$\boxed{\mathcal{L} = y^2 \left(\partial_a x^\mu - i\theta_i \sigma^\mu \partial_a \bar\theta^i + h.c.\right)^2}$$

Manifest $SU(4)$ (realized on θ^i), Q-susy, conformal invariance: same symmetries as in SYM

Quantum theory: 2-d conformal invariance for <u>any R</u>, must remain in (properly defined) $R \to 0$ limit

$\mathcal{L} = y^2 \partial x^\mu \partial x^\mu$: <u>not</u> conf. inv.: $\ln \varepsilon \int \frac{\partial y \partial y}{y^2}$

Divergence <u>cancelled</u> by <u>fermions</u> (l.c. gauge)

[$AdS_5 \times S^5$: $R_{mn} \sim g_{mn}$: cancelled by $(F_5 F_5)_{mn}$ from fermions separately for AdS_5 and S^5]

Central charge: $\int [dy^p]$ still in the measure, contribute to central charge

(Proper meaning of path integral ?

$$Z \sim \int [dy]\,[dS^5]\,[dx^r]\, \exp\left(-y^2 (\partial x - \theta\partial\theta)^2\right)$$

"4-d string with <u>fluctuating tension</u>"

$T \sim y^2 \equiv e^{2\varphi}$

Vertex operators ? $e^{s\varphi} h_{\mu_1 \dots \mu_s}(x) \partial x^{\mu_1} \dots \partial x^{\mu_s}_{+} \dots$

$\xrightarrow{?}$ $\partial^{2s} P_s h_s = 0$ at the boundary ($\varphi \to \infty$)

[expect string spectrum: $m = f(R, \alpha')$

$m\left(\frac{R}{\sqrt{\alpha'}} \to \infty\right) < \infty$, $m\left(\frac{R}{\sqrt{\alpha'}} \to 0\right) \to 0$]

Light Cone gauge phase space approach

Metsaev Thorn A.T. 00

$x^+ = \tau$

$y^2 \sqrt{g}\, g^{00} = -1$

} consistent gauge in AdS also Polyakov

Phase space formulation:

$h^{\mu\nu} \equiv \sqrt{g}\, g^{\mu\nu} y^2$, $x^+ = \tau$, $P^+ = p^+$

$h^{00} = -p^+$

$ds^2 = R^2 \left(y^2 \underbrace{dx_\mu dx_\mu}_{4} + \frac{1}{y^2} \underbrace{dy^m dy^m}_{6} \right)$

$x_\mu = (x_+, x_-, x_\perp)$, $(P^+, P^-, P_\perp)$

$\mathcal{H} = -P^- = \frac{1}{2p^+ \underset{\sim}{R}^2} \left(P_\perp^2 + \underset{\sim}{R}^4 y^4 x_\perp'^2 + \dots \right)$

$$\mathcal{L} = P_\perp \dot{x}_\perp + P_m \dot{y}^m + \tfrac{i}{2} p^+ \left(\theta^i \dot{\theta}_i + \zeta^i \dot{\zeta}_i + \text{h.c.}\right) - \mathcal{H}$$

$$\mathcal{H} = \frac{1}{2p^+R^2}\Big[P_\perp^2 + R^4 y^4 x_\perp'^2 + y^4 P_m^2 + R^4 y_m'^2$$

$$+ y^2\left(p^+ (\zeta^2)^2 + 2i p^+ \zeta \rho^{mn} \zeta y_m P_n\right)\Big]$$

$$- R^2 y y^m \left(\eta \rho^m \left(\theta' - i\sqrt{2}\, y\, \zeta\, x_\perp'\right) + \text{h.c.}\right)$$

$$p^+ x'^- + P_\perp x_\perp' + P_m y'^m + \tfrac{i}{2} p^+ \left(\theta^i \theta_i' + \zeta^i \zeta_i' + \text{h.c.}\right) = 0$$

Two options :

- direct $R \to 0$ limit,
  $p^+ R^2 = \tilde{p}^+ = \text{fixed}$
  6-d momentum part survives
  $x_\perp'^2$ goes away as in flat space
- $R \to 0$, $\tilde{y} = Ry = \text{fixed}$
  6-d momentum ($\tilde{P}_m = R^{-1} P_m$)
  dominates ?!

Solvable model ?

Higher spin massless modes ?

Hints of Weyl-invariant eqs. at the boundary are correct ?

Conclusions

- "Technical" relations between string theory and conformal supergravity
- Is there a more fundamental relation?
- Weyl-invariant field theory from some limit of string theory (in a special background)?
- Better understanding role of conformal sugra as induced in context of AdS/CFT (with ϵ) (unitarity not an issue in induced theory)
- $\lambda = \frac{R^2}{\alpha'} \to 0$ limit: conformal higher spins in 4-d, massless higher spins in AdS_5, derivation from string theory in $AdS_5 \times S^5$?