Conformal Supergravity and String theory

A. Tseytlin

String theory → supersymmetry → super gravity

$\alpha' \to 0$

2-d (Conformal) Supergravity → string theory

2-d

Deser, Zumino
Brink, di Vecchia, Howe
Brink, Schwarz
Polyakov
Fradkin, A.T.

4-d Quantum Gravity

String theory
(74, 82-84)

[extra massive states,
UV cutoff]

"R^2" gravity / conformal supergrv.
(77-84)

[extra massive ghosts]

4-d: Kaku, Townsend,
van Nieuwenhuizen
Bergshoeff, de Roo,
de Wit ...
Fradkin, A.T.

Relations between string theory
and conformal supergravity (98)

Ferrara et al
Liu, A.T.

AdS / CFT

gauged sugra in AdS_5 → conformal sugra in $R^{1,3}$,
induced by SYM, etc.

Generalization: conformal higher spins in $R^{1,3}$ ↔
massless higher spins in AdS_5

$\lambda \to 0$:

Witten
Sundborg
Vasiliev
Sezgin, Sundell

2-d Conformal sugra (coupled to matter):

NSR string

(1,1) : $(e_\mu^a, \chi_\mu) + (x^m, \psi^m)$ $m = 1, \dots, D$

$$\mathcal{L} = \sqrt{g}\Big(g^{\mu\nu}\,\partial_\mu x^m \partial_\nu x^m + i\bar{\psi}^m \gamma\cdot\partial\psi^m + (\partial_\mu x^m + \bar{\chi}_\mu \psi^m)\,\bar{\psi}^m \gamma^\rho \gamma^\mu \chi_\rho\Big)$$

diff. gauge : $g_{\mu\nu} = e^{2\rho}\delta_{\mu\nu}$, $\chi_\mu = \partial_\mu \chi$

Polyakov 81

Quantum theory:

$$Z \sim \int \mathcal{D}x\,\mathcal{D}\psi\; e^{-I}$$

Critical string: demand superconformal symmetry at quantum level $\to$ no anomaly no (ρ, χ) dependence induced

$$\ln Z \sim (D-10)\int d^2z\,\left(\partial\rho\,\partial\rho + i\bar{\chi}\,\partial\chi\right)$$

$\mapsto$ $D = 10$

2-d conformal sugra coupled to 10 supermultiplets is consistent at quantum level with same # of d.o.f. as at classical level

$\boxed{n = 2 \text{ string}}$: ("charged string")

Ademollo et al
Brink Schwarz

extended conformal supergravity

$$(e^a_\mu, \chi^i_\mu, A_\mu) + (\hat{x}^m, \hat{\psi}^m)$$

$\hat{x}^m = x^m + iy^m$
$(m = 1, \dots, D)$

$$\mathcal{L} = \sqrt{g}\Big[\, \partial \hat{x}^m \partial \hat{x}^{*m} + i\, \bar{\Psi}^m \gamma^\mu \mathcal{D}_\mu(A) \Psi^m$$
$$+ (\partial_\mu X^m + \bar{\chi}_\mu \Psi^m)\, \bar{\Psi}^m \gamma^\rho \gamma^\mu \chi_\rho + h.c. \Big]$$

$$\mathcal{D}\Psi = (\partial_\mu + \tfrac{1}{4}\omega_\mu + i A_\mu)\Psi + \bar{\chi}\gamma\chi\Psi$$

$$\chi = \chi^1 + i\chi^2$$

$$\delta e^a_\mu = \underline{\Omega}\, e^a_\mu + \dots$$
$$\delta \chi_\mu = i(\varepsilon - \underline{\gamma_5 \theta})\chi_\mu + \gamma_\mu \lambda + \dots$$
$$\delta A_\mu = \partial_\mu \varepsilon + \varepsilon_{\mu\rho} \partial^\rho \underline{\theta} + \dots$$

Classical gauge: $e^a_\mu = \delta^a_\mu$, $\chi_\mu = 0$, $A_\mu = 0$

Weyl + U(1) anomaly:

$e^a_\mu = e^{\underline{\rho}} \delta^a_\mu$, $\chi_\mu = \partial_\mu \underline{\chi}$, $A_\mu = \varepsilon_{\mu\nu} \partial^\nu \underline{\sigma}$

(ρ, σ, λ) - multiplet decouples for $D=2$

Fradkin, T. 81

$$\ln Z \sim (\mathcal{D}-2) \int d^2 z \left[(\partial\rho)^2 - (\partial\sigma)^2 + i\,\bar{\chi}\, \partial\!\!\!/\, \chi \right]$$

Simplification of extended conformal supergravity: find anomaly in U(1) (i.e. A_μ) sector

$$\int (\partial\rho)^2 \sim \int R\, \Box^{-1} R$$

$$\int (\partial\sigma)^2 \sim \int F_{\mu\nu}\, \Box^{-1} F_{\mu\nu} \sim \int (A^\perp_\mu)^2$$

induced (Schwinger) action: $\log\det \mathcal{D}\!\!\!/(A)$

Coefficient of $(A_\mu^\perp)^2 \sim (\partial\vec{\sigma})^2$ is easy to compute

pure conf. sugra : -2 matter : D

anomaly cancellation not possible for $n>2$ sugra

$n=4$ P.v.N., Sezgin

Conformal supergravity in 4 dimensions

Many analogies with 2-d case

N-extended conf. sugra + K vector multiplets
(SYM: dim $G = \kappa$)

Cancellation of anomalies - consistent (finite!) theory
for $\underset{\sim\sim\sim}{N=4}$, $\underset{\sim\sim\sim}{\kappa=4}$ only

Fradkin, A.T. 82-83

Absence of conformal and SU(4) anomalies necessary for quantum consistency (otherwise non-renormalizable at 2 loops)

1-loop ∞ or conf. anomalies only : $N=2$ non-renormaliz. theorem

Howe, Stelle, Townsend 84

$$T^\mu_\mu \sim b_4$$

$$\Gamma_\infty = \ln\Lambda \int d^4x \; b_4$$

Duff

$$\boxed{b_4 = \beta_1 R^* R^* + \beta_2 \left(W + \tfrac{1}{3}\partial^2 R\right)} \qquad W = R_{\mu\nu}^2 - \tfrac{1}{3}R^2 = \tfrac{1}{2}\left(C^2_{\dots} - R^* R^*\right)$$

$N=4$ vector multiplet $\quad (A_\mu, \Psi^i, X^{ij})$

$$\begin{array}{ccc} A_\mu, & \Psi^i, & X^{ij} \\ 1 & 4 & 6 \end{array}$$

$\beta_1 = 0\,,\quad \beta_2 = \frac{1}{2}$ (exact to all loops)

$g_{\mu\nu} = e^{2\rho}\,\tilde g_{\mu\nu}$: induced action $= \Gamma_{anom}[\rho,\tilde g] + \Gamma_{non\text{-}loc.}[\tilde g]$

Fradkin, A.T.
Riegert, 84

$$\Gamma_{anom} \sim \int d^4x\sqrt{\tilde g}\Big[\, (\beta_1 \tilde R^*\tilde R^* + \beta_2 \tilde W)\rho$$
$$+ (\beta_2 - 2\beta_1)\Big((\partial\rho\partial\rho)^2 + 2(\partial\rho)^2\tilde{\mathcal D}^2\rho + 2(\tilde R_{\mu\nu} - \tfrac{1}{2}\tilde g_{\mu\nu}\tilde R)\,\partial_\mu\rho\,\partial_\nu\rho\Big)$$
$$- \beta_2\left(\tilde{\mathcal D}^2\rho + \partial\rho\partial\rho - \tfrac{1}{6}\tilde R\right)^2 \Big] \sim \int \beta_2\,\rho\, D^4\rho + \ldots$$

$N=4$ SYM coupled to conformal supergravity

Bergshoeff, de Roo, de Wit
de Roo, Wagemans

$$(A_\mu, \Psi^i, X^{ij}) + \left(e^a_\mu, V_{\mu\,j}^{\;i}, \varphi, T^{ij}_{\mu\nu}, E_{ij}, \mathcal D^{ij}_{kl};\ \psi^i_\mu, \Lambda_i, \chi^k_{ij}\right)$$

$i,j = 1,2,3,4$
$SU(4)$

$$\varphi = C + i e^{-\phi}$$

$$\mathcal L_{SYM} \sim e^{-\phi} F_{\mu\nu}F^{\mu\nu} + C\, F_{\mu\nu}\overset{*}{F}{}^{\mu\nu} + \bar\Psi^i\gamma^\mu\mathcal D_\mu\Psi_i$$
$$+ X_{ij}\left(-\mathcal D^2 + \tfrac{1}{6}R\right)X^{ij} - X_{ij}F^{+\mu\nu}T^{ij}_{\mu\nu}$$
$$+ \mathcal D^{kl}_{ij}\left(X^{ij}X_{kl} - \tfrac{1}{6}\delta^i_k\delta^j_l |X|^2\right) + \ldots$$

$$F^+_{\mu\nu} = F_{\mu\nu} + \overset{*}{F}_{\mu\nu}$$

linear level = "current" × "field"

$$
\begin{aligned}
\mathcal{L}_{CSG} \sim\ & C_{\mu\nu\lambda\rho} C^{\mu\nu\lambda\rho} + \left(F^{i}_{j\,\mu\nu}(V)\right)^2 \\
& + \mathcal{D}^2\varphi^* \mathcal{D}^2\varphi - 2\left(R^{\mu\nu} - \tfrac{1}{3} g^{\mu\nu} R\right) \mathcal{D}_\mu \varphi^* \mathcal{D}_\nu \varphi \\
& + \mathcal{D}^\mu T^{ij+}_{\mu\nu} \mathcal{D}_\lambda T_{ij}^{\lambda\nu-} - \tfrac{1}{2} R^\mu_\nu T^{ij+}_{\mu\lambda} T_{ij}^{\lambda\nu-} \\
& + E^{ij}\left(-\mathcal{D}^2 + \tfrac{1}{6} R\right) E_{ij} + \mathcal{D}^{ij}_{kl} \mathcal{D}^{kl}_{ij} \\
& + \bar{\psi}^i_\mu \mathcal{D}^3_{\mu\nu} \psi_{\nu i} + \bar{\Lambda}^i \not{\mathcal{D}}^3 \Lambda_i + \bar{\chi}^k_{ij} \not{\mathcal{D}} \chi_k^{ij} + \dots
\end{aligned}
$$

Contributions to conformal anomaly:

	$h_{\mu\nu}$	ψ_μ	V_μ	$T_{\mu\nu}$	χ	Λ	E	φ
d.o.f.	6	−8	2	6	−2	−6	2	4
β_1	$\frac{137}{60}$	$-\frac{173}{180}$	$-\frac{13}{180}$	$\frac{11}{30}$	$\frac{7}{720}$	$\frac{21}{720}$	$\frac{1}{90}$	$\frac{1}{45}$
β_2	$\frac{199}{15}$	$-\frac{149}{30}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$-\frac{1}{60}$	$\frac{1}{30}$	$-\frac{4}{15}$

Field content and anomalies of N-extended CSG's:

$$
T^\mu_\mu \sim \beta_1 R^* R^* + \beta_2 \left(\tfrac{1}{2} C^2_{\lambda\mu\nu\rho} + \left(F^i_{j\mu\nu}\right)^2 + \dots\right)
$$

$U(N)$, $N = 1, 2, 3$; $SU(4)$

N	$h_{\mu\nu}$	ψ^i_μ	$V^i_{\mu\,j}$	$T^{ij}_{\mu\nu}$	χ^k_{ij}	Λ_i	E_{ij}	φ	β_1	β_2
1	1	1	1	0	0	0	0	0	$\frac{5}{4}$	$\frac{17}{2}$
2	1	2	4	1	2	0	0	0	$\frac{11}{24}$	$\frac{13}{3}$
3	1	3	9	3	9	1	3	0	0	1
4	1	4	15	6	20	4	10	1	0	−2

N=4 CSG + SYM (dim G = κ) :

$$\beta_1 = 0 + 0 = 0 , \qquad \beta_2 = \tfrac{1}{2}\cdot\kappa - 2$$

(analogy with 2-d case!)

$$\boxed{\kappa = 4}$$

- same result for coeffs of $F^2_{\mu\nu}(V)$ (simpler!)
- same result from cancellation of $SU(4)$ chiral anomaly

$$\partial_\mu J^\mu_5 \sim F_{\mu\nu}(V)\, F^*_{\mu\nu}(V)$$

Römer, van Nieuwenhuizen 85

Consistent with conformal and axial anomaly being in one multiplet

Correspondence with Einstein theory at low energies?

Fujii, ... Kaku, Townsend 78

break conformal invariance spontaneously :

one vector multiplet (ghost - like)

$$\mathcal{L} = X\left(-\partial^2 + \frac{R}{6}\right)X + \dots , \qquad \langle X \rangle = a$$

$$\mathcal{L} = \frac{a^2}{6} R + \dots$$

(no Λ-term if flat directions in potential)

Natural in finite superconformal theory with scalars

Fradkin, T. Strominger Nair 84

Equivalent to making non-anomalous Weyl transformation (in phase with broken symm.)

$$|X_{ij}|' = \text{const} = a , \qquad g'_{\mu\nu} = a^2/|X_{ij}|^2\, g_{\mu\nu} \quad \dots$$

Issue of ghosts / unitarity

Euclidean path integral is well-defined (without ghost multiplets)

Conformal supergravity as "induced" theory : AdS / CFT context

$$Z[G] = e^{-\Gamma[G]} = \int dA \; e^{-S[A,G]}$$

$A = (A_\mu, \psi^i, X^{ij})$ SYM $SU(N)$

$G = (g_{\mu\nu}, \phi, C, \psi^i_\mu, \dots)$ conformal sugra background

$$S_{int} = \int d^4x \left[T_{\mu\nu}(A)\, h_{\mu\nu} + F^2_{\mu\nu}(A)\, \phi + \dots \right]$$

Z not superconf. inv. : anomaly

$$\langle T^\mu_\mu \rangle = \frac{2 g_{\mu\nu}}{\sqrt{g}} \frac{\delta \Gamma}{\delta g_{\mu\nu}} = -\beta\, \mathcal{L}_{CSG}$$

$$\mathcal{L}_{CSG} = C^2_{\lambda\mu\nu\rho} - R^* R^* + 2\left(F_{\mu\nu}{}^i{}_j(V)\right)^2 + 4\phi \Delta_4 \phi + \dots$$

$$\mathcal{D}_\mu \frac{\delta \Gamma}{\delta V_\mu} = \beta\, F_{\mu\nu}(V)\, F^*(V)^{\mu\nu}$$

$$\beta = \frac{N^2}{4(4\pi)^2}$$

$$\Gamma = \Gamma_\infty + \Gamma_{fin} \quad , \quad \Gamma_{fin} = \Gamma_{anom} + \Gamma_{inv}$$

$$\Gamma_\infty = -\beta \ln \Lambda \underline{\int d^4x \sqrt{g}\, \mathcal{L}_{CSG}} \qquad g_{\mu\nu} = e^{2\rho} \tilde{g}_{\mu\nu}$$

$$\Gamma_{anom} \sim \beta \int \Big[\left(\tilde{C}^2 - \tilde{R}^* \tilde{R}^* + \dots \right) \rho + \dots$$
$$+ F^{\mu\nu}(V)\, F^*_{\mu\nu}(V)\, \mathcal{D}^{-2} \mathcal{D}^\lambda V_\lambda + \dots \Big]$$

Quadratic approximation:

$$\Gamma_2 = \beta \int \Big[C^{\mu\nu\lambda\rho} \ln\left(\frac{-\partial^2}{\Lambda^2}\right) C_{\mu\nu\lambda\rho} + 2 F^{\mu\nu} \ln\left(-\frac{\partial^2}{\Lambda^2}\right) F_{\mu\nu} + 4 \partial^2\phi \ln\left(-\frac{\partial^2}{\Lambda^2}\right) \partial^2\phi + \ldots \Big]$$

$$\frac{\delta^n \Gamma}{\delta h_{\mu\nu}^{(x_1)} \ldots \delta h_{\alpha\beta}^{(x_n)}} \sim \langle T_{\mu\nu}^{(x_1)} \ldots T_{\alpha\beta}^{(x_n)} \rangle$$

conformal inv. (at separated points)

AdS/CFT

$N \to \infty$, $\lambda = g_{YM}^2 N \gg 1$: $D=5$, $N=8$ gauged sugra

$$\Gamma[G] = I_{GSG}[\hat{G}(G)] \qquad \hat{G}\big|_{\partial AdS} = G$$

solution of Dirichlet problem

Quadratic terms:

$$\int d^5x \sqrt{g}\, (\partial\phi)^2 \to \int d^4x\, \phi\, \partial^4 \ln -\frac{\partial^2}{\Lambda^2} \phi$$

Gubser Klebanov Polyakov

$$\int d^5x \sqrt{g}\, F_{mn}^2 \to \int d^4x\, F^{\mu\nu} \ln -\frac{\partial^2}{\Lambda^2} F_{\mu\nu}$$

Witten

$$\int d^5x \sqrt{g}(\hat{R} - 2\lambda) \to \int d^4x\, \partial^2 \bar{h}^{\perp}_{\mu\nu} \ln -\frac{\partial^2}{\Lambda^2} \partial^2 \bar{h}^{\perp}_{\mu\nu} + \ldots$$

Liu, A.T.

$$= 2\int C^{\mu\nu\lambda\rho} \ln -\frac{\partial^2}{\Lambda^2} C_{\mu\nu\lambda\rho} + \ldots$$

AdS_{d+1}:

$$ds^2 = \frac{R^2}{z^2}(dz^2 + d\vec{x}^2)$$

$$S \sim \int d^{d+1}x \sqrt{g}\, (\partial\phi\partial\phi + m^2\phi^2), \qquad \phi\big|_{\partial M} = \phi_0 \quad (z \to 0)$$

$$\phi(z,x) = c \int d^d x' \; \frac{z^{\Delta_+}}{(z^2 + |x-x'|^2)^{\Delta_+}} \; \phi_0(x')$$

Witten
Friedman
et al

$$\Delta_+ = \frac{1}{2}\left(d \pm \sqrt{d^2 + 4\mu^2}\right) \,, \qquad \mu \equiv mR$$

$$S \sim \int d^d x \, d^d x' \; \frac{\phi_0(x) \; \phi_0(x')}{(\varepsilon^2 + |x-x'|^2)^{\Delta_+}}$$

$\varepsilon \sim \Lambda^{-1}$
$(z = \varepsilon \to 0)$

Graviton: $\hat{h}_{zz}\big|_{z=0} = \hat{h}_{z\mu}\big|_{z=0} = 0 \;, \quad \hat{h}_{\mu\nu}\big|_{z=0} \sim h_{\mu\nu}(x)$

$$S \sim \int d^d x \, d^d x' \; \frac{h_{\mu\nu}(x') \; H_{\mu\nu\lambda\rho}(x-x') \; h_{\lambda\rho}(x')}{(\varepsilon^2 + |x-x'|^2)^d}$$

$$H_{\mu\nu\lambda\rho} \sim (\delta_{\mu(\lambda}\delta_{\nu)\rho} - \text{trace}) - \frac{1}{x^2}\left(\delta_{\nu\lambda} x_\mu x_\rho + 3 \text{ terms}\right) + \frac{4}{x^4} x_\mu x_\nu x_\lambda x_\rho$$

(same as in $\langle T_{\mu\nu}(x) \; T_{\lambda\rho}(x') \rangle$ in a CFT)

Similar relations for higher-order invariant and anomalous terms in Γ and $d=5$ sugra
(CS in $d=5$: $\int V \wedge dV \wedge dV \;\to\; \int F F^* \frac{1}{\partial^2} \partial^m V_m$)

Generalizations ?

Finite cutoff $(z = \varepsilon \neq 0)$: dynamical gravity*
on the brane at $z = \varepsilon$ $\to$ conformal sugra fields should be integrated over? (true "induced CSG")
(cf. Randall - Sundrum)

Balasubramanian,
Gimon, Minic,
Rahmfeld

(* Sugra fields (non-normalizable modes) have ∞ action if bulk is infinite but will have to fluctuate in finite bulk)

How to justify from string theory perspective ?

Higher-spin generalization

Free <u>conformal</u> "pure spins" in flat space $(d=4)$

$$\mathcal{L} = \sum_s \left(\varphi_s \, \partial^{2s} P_s \, \varphi_s \; + \; \psi_{s-\frac{1}{2}} \partial^{2s-1} \not\partial \, P_{s-\frac{1}{2}} \psi_{s-\frac{1}{2}} \right)$$

(Weyl-inv operators) Fradkin A.T. 84

$s = 2, 3/2, 1$: conformal supergravity $\{ \varphi_{\mu_1 \ldots \mu_s}, \psi_{\mu_1 \ldots \mu_{s-1/2}} \}$

P_s : transverse, traceless; totally symmetric $P_s^2 = P_s$

Dimensionless coupling ; interaction not prohibited in $R^{1,3}$

higher derivative - non-unitary ; superalgebras & cubic interactions Fradkin Linetsky 91

Relation to higher massless spins in AdS_5

in same way as $CSG_4 \leftarrow GSG_5$

"Induced theory" interpretation:

Free $N=4$ vector multiplet (or <u>$\lambda = 0$</u> SYM):

<u>conserved</u> <u>traceless</u> <u>higher spin</u> <u>currents</u> Burgess et al Anselmi. Konshtein Vasiliev Zaikin

$$J_s \sim X \, P_s \, \partial^s X + \ldots$$

← scalar of vector multiplet

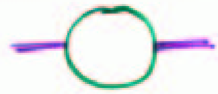
Couple to φ_s :

$$\mathcal{L} = (X \partial^2 X + \ldots) + \varphi_s(x) \, X \, P_s \, \partial^s X + \ldots$$

Integrate out vector multiplet fields →

<u>induced action</u> for $\varphi_s, \psi_s, \ldots$ (cf. conformal sugra case)

$$\ln \det \left(\partial^2 + \varphi_s(x) P_s \partial^s \right)$$



$$\to \int \varphi_s \left(\frac{\partial^s}{\partial^2} P_s \right)^2 \varphi_s + \dots \sim \int \varphi_s \, \partial^{2s} \ln\left(-\frac{\partial^2}{\Lambda^2}\right) P_s \varphi_s + \dots$$

$$\boxed{\Gamma_\infty \sim \ln\Lambda \int \varphi_s \, \partial^{2s} P_s \, \varphi_s}$$

$$\Gamma_\lambda \sim \int d^4x \, d^4x' \; \varphi_s(x) \frac{\hat{P}_s(x-x')}{(\epsilon^2 + |x-x'|^2)^{s+2}} \varphi_s(x')$$

Should follow from solution of Dirichlet problem for higher-spin massless field in AdS_5 : $\int (\hat{\varphi}_s \, \mathcal{D}^2 \varphi_s + \dots)$ [agreement guaranteed by symmetry considerations*]

$\lambda \to 0$ limit of AdS/CFT

Witten Sundborg 00'

Quantum 4-d vector multiplet theory : way to reconstruct interactions of φ_s ? (CSG case-prototype)

non-anomalous part of Γ depends only on original pure spin d.o.f.

Why there are such states in spectrum of superstring in $AdS_5 \times S^5$ in $\lambda \to 0$?

Metsaev 00'

* Light cone gauge description $\varphi_s \to \{ \varphi_{s'} \}$

$$\int d^{d+1}x \; \varphi_{s'} \left(\partial^2 - \frac{1}{z^2} M_{s'} \right) \varphi_{s'} , \quad M_{s'} = \left(s' + \frac{d-4}{2} \right)^2 - \frac{1}{4}$$

"Zero Tension" limit of superstring in $AdS_5 \times S^5$

dimensionless parameters: $\boxed{\sqrt{\lambda} = \frac{R^2}{\alpha'}}$, $\alpha' p_i^2$, g_s

$\lambda \to 0$: non-trivial limit (cf. flat space)
($R \to 0$ or $\alpha' \to \infty$)

Special property of AdS_{d+1} space (cf. sphere):

$$\mathcal{L} = \partial X^M \partial X^M + \Lambda \left(X^M X_M - \sqrt{\lambda} \right)$$

$$X_M X^M = X_{d+1}^2 + X_\mu X_\mu - X_{d+2}^2$$

Sphere: $X^M X_M \geq 0$: $\lambda \to 0 \to X_M = 0$

AdS : $\lambda \to 0$: $X^M X_M = 0$

"string moving on cone in d+2 dim"

Reduction by 2 dimensions: $\tau = \varphi \pm \psi$, $r = e^\varphi$, $v = e^\psi$

$X_{d+1} \pm X_{d+2} = u, v$, solve for u; $r^2 \equiv X_i \cdot X_i$

$$\mathcal{L} = r^2 \left(\partial\tau \partial\tau + (\partial \Omega_{d-1})^2 \right) \quad : R \times S^{d-1}$$

Same in Poincare coordinates: $\tilde{y} = \lambda^{1/4} y$

$$\mathcal{L} = \sqrt{\lambda} \left(\frac{\partial y \partial y}{y^2} + y^2 \partial x_\mu \partial x_\mu \right) \to \boxed{\tilde{y}^2 \partial x_\mu \partial x_\mu}$$

$T \to 0$ limit in flat space

$\alpha' \to \infty$: $\quad \mathcal{L} \sim \frac{1}{\alpha'} \partial x \partial x + \int e^{ipx} \ldots$ Schild

Lindstrom
Karlhede

$$x \to \sqrt{\alpha'}\, x \quad , \quad p \to \frac{1}{\sqrt{\alpha'}}\, p$$

momenta $\to \infty$ for $\alpha' \to \infty$

UV limit : space – time conformal invariance ?

$$\mathcal{H} \sim \alpha' P^2 + \cancelto{0}{\frac{1}{\alpha'} x'^2} \to \alpha' P^2$$

decoupled particles

$$\mathcal{L} = T \sqrt{\det \partial x \partial x} \longrightarrow \chi \det \partial x \partial x + T^2 \chi^{-1}$$

$$\to \chi \det \partial x \partial x \quad \text{or} \quad \chi \cdot \left(\sqrt{g}\, \partial x \partial x \right)^2$$

Lindstrom et al

χ – auxiliary field

Quantum theory ? Only massless states ?

Curved space / AdS case is different:

$\frac{R^2}{\alpha'} \to 0$ but momenta are fixed

$$\chi (\partial x \partial x)^2 \longrightarrow y^2 \partial x \partial x$$

radial direction of AdS

Bosonic part of $AdS_5 \times S^5$ action:

S^5 and kinetic term of y of AdS_5 decouple

$$\mathcal{L} = y^2 \, \partial x^\mu \partial x^\mu$$ (Naive limit!)

y, x^μ independent : $\int \mathcal{D}y \, \mathcal{D}x \; e^{-I[y,x]}$

Suggests Weyl invariance at boundary

$$\mathcal{L} = y^2 \, g_{\mu\nu}(x) \, \partial x^\mu \partial x^\nu$$

$$g_{\mu\nu} \to K^2(x) g_{\mu\nu}, \qquad y \to K^{-1}(x)\, y .$$

(only conformal symmetry if $\frac{\partial y \, \partial y}{y^2}$ is present)

→ Weyl invariance of eqs of states → conformal higher spins at the boundary ?!

Naive limit: conformal invariance ?
central charge ?
take limit in correlation functions?

Including fermions:

Metsaev, A.T. 98

Covariant κ-symmetry gauge : $\eta^i = 0$

$PSU(2,2|4)$: $(Q_i, S_i) \to (\theta_i, \eta_i)$ $\quad i = 1,2,3,4$ $\quad SU(4)$

$$ds^2 = y^2 \, dx_r \, dx_r + \frac{dy^p \, dy^p}{y^2}$$ $(p = 1,2,\dots,6)$ $\quad r = 0,1,2,3$

γ-matrices: (σ^μ, ρ^P) $\quad$ 4+6 split

$$\mathcal{L} = \frac{R^2}{\alpha'}\Big[\, y^2 \left(\partial_a x^\mu - i\,\theta_i\, \sigma^\mu \partial_a \bar{\theta}^i + h.c.\right)^2 + \underbrace{y^{-2}\, \partial y^\rho \partial y^\rho}_{\to 0} - \underbrace{i\, \epsilon^{ab}\, \partial_a y^\rho\, \bar{\theta}^i \rho^\rho_{ij}\, \theta^j + h.c.}_{\to 0} \Big]$$

$$y \to \left(\frac{R^2}{\alpha'}\right)^{-1/2} y\,, \qquad \frac{R^2}{\alpha'} \to 0 \quad :$$

$$\boxed{\mathcal{L} = y^2 \left(\partial_a x^\mu - i\,\theta_i\, \sigma^\mu \partial_a \bar{\theta}^i + h.c.\right)^2}$$

Manifest $SU(4)$ (realized on θ^i), Q-susy, conformal invariance: same symmetries as in SYM

Quantum theory: 2-d conformal invariance for <u>any</u> R, must remain in (properly defined) $R \to 0$ limit

$\mathcal{L} = y^2 \partial x^\mu \partial x^\mu$: <u>not</u> conf. inv.:

$\ln \epsilon \; \dfrac{\partial y \partial y}{y^2}$

Divergence <u>cancelled</u> by <u>fermions</u> (l.c. gauge)

[$AdS_5 \times S^5$: $R_{mn} \sim g_{mn}$: cancelled by $(F_5 F_5)_{mn}$ from fermions separately for AdS_5 and S^5]

Central charge: $\int [dy^\rho]$ still in the measure, contribute to central charge

(Proper meaning of path integral ?

$$Z \sim \int [dy] \int [dS^5] \int [dx^\mu] \;\; \exp\left(-y^2 (\partial x - \theta \partial \theta)^2\right)$$

"4-d string with fluctuating tension"

$$T \sim y^2 \equiv e^{2\varphi}$$

Vertex operators ? $\quad e^{s\varphi} h_{\mu_1 \cdots \mu_s}(x)\, \partial x^{\mu_1} \ldots \partial x^{\mu_s} + \ldots$

$$\xrightarrow{?} \quad \partial^{2s} P_s h_s = 0 \quad \text{at the boundary } (\varphi \to \infty)$$

[expect string spectrum: $\quad m = f(R, \alpha')$

$m\left(\frac{R}{\sqrt{\alpha'}} \to \infty\right) < \infty \,, \qquad m\left(\frac{R}{\sqrt{\alpha'}} \to 0\right) \to 0$]

Light Cone gauge phase space approach

Metsaev, Thorn, A.T. 00

$$\left.\begin{aligned} x^+ &= \tau \\ y^2 \sqrt{g}\, g^{00} &= -1 \end{aligned}\right\}$$

consistent gauge in AdS

also Polyakov

Phase space formulation:

$$h^{\mu\nu} \equiv \sqrt{g}\, g^{\mu\nu} y^2 \,, \qquad x^+ = \tau, \quad P^+ = p^+$$

$$h^{00} = -p^+$$

$$ds^2 = R^2 \Big(y^2 \underbrace{dx_\mu dx_\mu}_{4} + \frac{1}{y^2} \underbrace{dy^m dy^m}_{6} \Big)$$

$$x_\mu = (x_+, x_-, x_\perp) \,, \qquad (P^+, P^-, P_\perp)$$

$$\mathcal{H} = -P^- = \frac{1}{2p^+ R^2} \Big(P_\perp^2 + R^4 y^4 x_\perp'^2 + \ldots \Big)$$

$$\mathcal{L} = P_\perp \dot{x}_\perp + P_m \dot{y}^m + \tfrac{i}{2} p^+ \left(\theta^i \dot{\theta}_i + \eta^i \dot{\eta}_i + h.c. \right) - \mathcal{H}$$

$$\mathcal{H} = \frac{1}{2p^+R^2} \Big[\underbrace{P_\perp^2 + R^4 y^4 x_\perp'^2} + \underbrace{y^4 P_m^2 + R^4 y_m'^2}$$

$$+ \; y^2 \left(p^+ (\eta^2)^2 + 2i\, p^+ \eta \rho^{mn} \eta\, y_m P_n \right) \Big]$$

$$- R^2 y y^m \left(\eta \rho^m \left(\theta' - i\sqrt{2}\, y\, \eta\, x_\perp' \right) + h.c. \right)$$

$$p^+ x'^- + P_\perp x_\perp' + P_m y'^m + \tfrac{i}{2} p^+ \left(\theta^i \theta_i' + \eta^i \eta_i' + h.c. \right) = 0$$

Two options :

- direct $R \to 0$ limit,
  $p^+ R^2 = \tilde{p}^+ =$ fixed
  6-d momentum part <u>survives</u>
  $x_\perp'^2$ goes away as in flat space
- $R \to 0$, $\tilde{y} = Ry =$ fixed
  6-d momentum $(\tilde{P}_m = R^{-1} P_m)$
  dominates ?!

Solvable model ?

Higher spin massless modes ?

Hints of Weyl-invariant eqs. at the boundary are correct ?

Conclusions

- "Technical" relations between string theory and conformal supergravity
- Is there a more fundamental relation?
- Weyl-invariant field theory from some limit of string theory (in a special background)?
- Better understanding role of conformal sugra as induced in context of AdS/CFT (with ε) (unitarity not an issue in induced theory)
- $\lambda = \frac{R^2}{\alpha'} \to 0$ limit: conformal higher spins in 4-d, massless higher spins in AdS_5, derivation from string theory in $AdS_5 \times S^5$?