AdS/CFT correspondence tests in four and six dimensions

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I. The rôle of BPS operators in the AdS/CFT correspondence

One of the most fascinating developments of the recent years, in which Supergravity plays a central rôle, is the AdS/CFT conjecture. It predicts the correspondence between KK states in Type IIB supergravity on, for example, $\mathrm{AdS}_{5/7} \times S^{5/3}$ and 1/2 BPS short gauge invariant operators in the boundary $\mathrm{SCFT}_{4/6}$ of $N = 4$ $D = 4$ super-Yang-Mills (SYM) or of the $N = (2, 0)$ $D = 6$ self-dual tensor multiplet.

BPS operators are very special representations of the superconformal symmetry underlying both the AdS and SCFT pictures. Their main property is that the symmetry fixes their conformal dimension. For this reason they are called protected operators. The simplest example is the stress-tensor multiplet in these two SCFT's.

Most efforts so far have been devoted to testing this correspondence by computing and comparing correlation functions of BPS operators in SCFT to their supergravity counterparts in $D = 4$. The main results take the form of non-renormalization theorems for various correlators of 1/2 BPS operators. These include:

-Non-renormalization of 2- and 3-point functions Lee, Minwalla, Rangamani, Seiberg (1998); Freedman, Mathur, Matusis, Rastelli (1999); Arutyunov, Frolov (1999); D'Hoker, Freedman, Skiba (1999); Howe, E.S., West (1998); Intriligator, Skiba (1999); Eden, Howe, West (1999); Bianchi, Kovacs, Rossi, Stanev (1999); Penati, Santambrogio, Zanon (1999); D'Hoker, Ryzhov (2001)

-Non-renormalization of n–point functions of "extremal" and "subextremal" types D'Hoker, Freedman, Mathur, Matusis, Rastelli (1999); Eden, Howe, Schubert, E.S., West (2000); Erdmenger, Perez-Victoria (2000)

More elaborate tests involve 4-point functions which are renormalized. The typical example is the correlator of 4 stress-tensor multiplets in $D = 4$ $N = 4$ SYM. Different methods have been employed:

-supergravity calculations Liu, Tseytlin (1999); D'Hoker, Freedman (1999); Freedman, Mathur, Matusis, Rastelli (1999); Arutyunov, Frolov (2000)

-explicit perturbative and instanton calculations Eden, Howe, Schubert, E.S., West (1999); Gonzalez-Rey, Park, Schalm (1999); Bianchi, Kovacs, Rossi, Stanev (1999-2000); Eden, Schubert, E.S. (2000)

-analysis of the superconformal invariants undergoing BPS shortening Howe, West (1996-1999); Eden, Howe, Schubert, E.S., West (1999-2000); Eden, Howe, Pickering, E.S., West (2000); Bianchi, Kovacs, Rossi, Stanev (1999,2000); Eden, Schubert, E.S. (2000) This lead to the "partial non-renormalization" theorem for the 4-point function of stress-tensor multiplets Eden, Petkou, Schubert, E.S. (2000)

-study of the OPE of two stress-tensor or more general 1/2 BPS short multiplets. This lead to the discovery of a new protection mechanism, different from BPS shortening Arutyunov, Frolov, Petkou (2000); Arutyunov, Eden, Petkou, E.S. (2001); Eden, E.S. (2001); Eden, Ferrara, E.S. (2001); Ferrara, E.S. (2001); Heslop, Howe (2001); Bianchi, Kovacs, Rossi, Stanev (2001); Penati, Santambrogio (2001)

In this talk I review the main ideas of the study of the correlator of 4 stress-tensor multiplets in $D = 4$ $N = 4$ SYM and its comparison to the AdS supergravity calculations. I also report some new results on the $D = 6$ $N = (2,0)$ tensor multiplet, both on the CFT and AdS sides of the correspondence.

II. What is a 1/2 BPS short operator?

The superconformal algebras PSU(2,2/4) (for $D = 4$) and OSp(8^*/4) (for $D = 6$) which underlie the corresponding maximal SCFT and AdS supergravity admit special representations which are annihilated by one half of the Poincaré SUSY generators:

$$Q^{+}_{\alpha}\,|\Phi\rangle = 0 \qquad \begin{array}{l} + \text{ is a charge of a U(1) subgroup} \\ \text{of the R symmetry group } G \end{array}$$

as well as by all the raising operators of G:

$$L^{++}\,|\Phi\rangle = 0\,; \qquad L^{++},\ L^{0},\ L^{--} \text{ is the Cartan decomposition of } G$$

These two conditions are only compatible for special, quantized values of the conformal dimension of $|\Phi\rangle$ depending on its spin and R symmetry quantum numbers $\Rightarrow$ the BPS states have protected dimension.

The most familiar, but by far not the only example is the $D = 4$ $N = 1$ chiral superfield:

$$\bar{D}_{\dot\alpha}\Phi(x,\theta,\bar\theta) = 0 \ \Rightarrow\ \Phi = \Phi(x,\theta)$$

This has lead to the widespread but improper use of the term Chiral Primary Operator: Chirality is just one particular type of $D = 4$ BPS shortening; it does not exist in $D = 3,5$ whereas everything is chiral in $D = 6$.

Realization in Harmonic Superspace Galperin, Ivanov, Kalitsyn, Ogievetsky, E.S. (1984); Hartwell, Howe (1995):

Consider superfields $\Phi(x, \theta, u)$ depending on the extra bosonic coordinates of the

$$\text{Harmonic coset} \qquad \{u\} \sim \frac{G}{[U(1)]^{\text{rank}(G)}}$$

Then the two 1/2 BPS defining conditions are translated into differential constraints:

$$D^+_\alpha\, \Phi(x, \theta, u) = 0 \;\Rightarrow\; \begin{array}{l}\text{Grassmann analyticity:}\\ \Phi = \Phi(x, \theta^+, u)\end{array}$$

$$D^{++}_u\, \Phi(x, \theta, u) = 0 \;\Rightarrow\; \begin{array}{l}\text{Harmonic analyticity:}\\ \Leftrightarrow \text{irreducibility under } G\end{array}$$

Here are some typical examples:

- $D = 4$ $N = 2$ hypermultiplet $q^+(x, \theta^+, \bar\theta^+, u)$
- $D = 4$ $N = 4$ SYM field strength $W^{12}(x, \theta^{1,2}, \bar\theta_{3,4}, u)$
- $D = 6$ $N = (2, 0)$ tensor multiplet $W^{12}(x, \theta^{1,2}, \bar\theta_{3,4}, u)$

Analyticity implies field equations for the physical fields.

Crucial property: analytic SF's form a ring structure $\Rightarrow$
Composite analytic (BPS) operators Howe, West (1995-1997); Andrianopoli, Ferrara (1998); Andrianopoli, Ferrara, E.S., Zupnik (2000); Ferrara, E.S. (2000); Heslop, Howe (2000)

For example, bilinear composites ("supercurrents"):

- $D = 4$ $N = 2$ R-current multiplet

$$\mathcal{J}^{++} = \tilde q^+ q^+ = \cdots + \theta^+ \sigma^\mu \bar\theta^+\, j_\mu(x) + \cdots$$

- $D=4\ N=4$ stress-tensor multiplet

$$\mathcal{T}^{1122} = W^{12}W^{12} = \cdots + \theta^{[1}\sigma^{\mu}\bar{\theta}_{[3}\ \theta^{2]}\sigma^{\nu}\bar{\theta}_{4]}\ T_{\mu\nu}(x) + \cdots$$

- $D=6\ N=(2,0)$ stress-tensor multiplet $\mathcal{T}^{1122} = W^{12}W^{12}$

Analyticity implies conservation for the bilinear composites.

III. Four-point stress-tensor correlators in D=4, 6

Special property of the 4-point function of 1/2 BPS operators:
All the higher components of the correlator $\langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\rangle$ are uniquely fixed by CSUSY, starting from its lowest ($\theta=0$) component ("superconformal primary").

We shall concentrate on the new case $D=6$, the case $D=4$ being very similar. The lowest component is determined by the R symmetry USp(4)$\sim$SO(5) and by conformal invariance:

$$D=6: \qquad \langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\rangle|_{\theta=0}$$

$$= a_1(s,t)\frac{\delta^{I_1I_2}\delta^{I_3I_4}}{x_{12}^8x_{34}^8} + a_2(s,t)\frac{\delta^{I_1I_3}\delta^{I_2I_4}}{x_{13}^8x_{24}^8} + a_3(s,t)\frac{\delta^{I_1I_4}\delta^{I_2I_3}}{x_{14}^8x_{23}^8}$$

$$+b_1(s,t)\frac{C_{I_1I_3I_2I_4}}{x_{13}^4x_{14}^4x_{23}^4x_{24}^4} + b_2(s,t)\frac{C_{I_1I_2I_3I_4}}{x_{12}^4x_{14}^4x_{23}^4x_{34}^4} + b_3(s,t)\frac{C_{I_1I_2I_4I_3}}{x_{12}^4x_{13}^4x_{24}^4x_{34}^4}$$

where I is an SO(5) vector index, $C_{I_1I_2I_3I_4}$ is a SO(5) invariant tensor and

$$s = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2} \qquad t = \frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}$$

are the conformal cross-ratios.

Crossing symmetry reduces the 6 functions a_i and b_i to only two independent coefficients:

$$a_3(t,s) = a_1(s,t) \qquad b_2(s,t) = b_2(t,s)$$

Next, we have to impose the 1/2 BPS conditions (analyticity):

Grassmann Analyticity (dependence on half of the θ's) is made manifest by constructing the corresponding unique OSp$(8^*/4)$ invariant.

Harmonic Analyticity (absence of harmonic singularity) is the most non-trivial part. Following the method of Eden, Howe, Pickering, E.S., West (2000), we obtain our main result :

Constraints on the $D = 6$ 4-point amplitude:

$$\partial_t b_2 = \frac{s^2}{t^2}\partial_s a_3 - \frac{t}{s}\partial_s a_1 - \frac{t(s+t-1)}{s^2}\partial_t a_1$$
$$\partial_s b_2 = \frac{t^2}{s^2}\partial_t a_1 - \frac{s}{t}\partial_t a_3 - \frac{s(s+t-1)}{t^2}\partial_s a_3$$

The integrability condition for these coupled 1st-order PDE's

$$t\Delta\left(\frac{a_1}{s^2}\right) = s\Delta\left(\frac{a_3}{t^2}\right) \ ,$$

$$\Delta = s\partial_{ss} + t\partial_{tt} + (s+t-1)\partial_{st} + 3\partial_s + 3\partial_t$$

is in fact equivalent to the Ward identity

$$\partial_\mu \langle j^\mu(x_1)\bar\phi\phi(x_2)\bar\phi\phi(x_3)\bar\phi\phi(x_4)\rangle = 0$$

on the component correlator of one R current and three scalar bilinears.

Conclusion: The 1st-order constraints on the amplitude are stronger than the standard Ward identities for the R currents or the stress-tensor and have no direct analog in familiar QFT.

III. Comparison to AdS supergravity calculations

$D = 4$

The pioneering work of D'Hoker, Freedman et al gave the 4-point function of dilatons and axions in AdS$_5$ supergravity. It should match the correlator of 4 stress-tensors in $N = 4$ SYM $\rightarrow$ difficult to check (8 derivatives of the lowest component of the supercorrelator).

Arutyunov and Frolov pushed this non-trivial AdS calculation further and found the term which should directly match the correlator of 4 scalar bilinears in SCFT$_4$.

Comparing the AdS$_5$ and SCFT$_4$ results, one finds an even stronger constraint:

All the six coefficients of the quantum corrections to the amplitude are given by a single symmetric function $\mathcal{F}(s,t)$:

$$a_1 = s\mathcal{F}(s,t), \quad a_3 = t\mathcal{F}(s,t), \quad b_2 = (1-s-t)\mathcal{F}(s,t) \quad \text{and crossing}$$

Explanation: In $D = 4$ we have further dynamical input from QFT via Intriligator's insertion procedure:

$$\frac{\partial}{\partial g^2}\langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\rangle \sim \int_5 \langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\mathcal{L}(5)\rangle$$

where $\mathcal{L}(5)$ is the interacting Lagrangian of the $N = 4$ theory (strictly speaking, this only applies to the $N = 2$ off-shell formulation of the $N = 4$ theory).

The 5-point superspace invariant $\langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\mathcal{L}(5)\rangle$ is unique up to a space-time dependent factor $\rightarrow$ becomes $\mathcal{F}(s,t)$ after integration over x_5.

This result has been confirmed by all available perturbative (1- and 2-loop), instanton and strong coupling (AdS) calculations. The general "partial non-renormalization" theorem was proved in Eden, Petkou, Schubert, E.S. (2000).

$D = 6$

This is a new result (Arutyunov and E.S., to appear in a few days ...). The AdS_7 calculation has just been completed, the coefficients a_1, a_3 and b_2 have been expressed in terms of the familiar D-functions and the 1st-order PDE's have been confirmed.

Note that Intriligator's procedure does not apply in $D = 6$ because of lack of interacting field theory.

IV. Conclusion

The AdS/CFT correspondence has successfully passed a new non-trivial test in a situation where no weak coupling limit exists.

The Big Open Question: We understand very well the origin of such differential constraints on the CFT side (Harmonic Analyticity). What is the corresponding feature of the AdS picture?

This result should hold not only for the supergravity amplitude but also for all stringy corrections.

We need a better understanding of brane theory!