HIGHER ORDER INVARIANTS IN SUPERGRAVITY

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STONY BROOK 2001

STONY BROOK 1979

3-LOOP
- DESER, KAY, STELLE '77
- FERRARA, VAN NIEUWENHUIZEN '78

$$L^{(n)} \sim \kappa^{2(n-1)} R^{n+1}$$

$$\int d^6x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda} \not{\partial} \lambda \right]$$

$$\int d^6x \left[J_\mu J^\mu + \mathcal{D}_\mu \mathcal{D}^\mu + \tfrac{i}{2} \bar{J}_\mu \not{\partial} J^\mu - \tfrac{3}{4} C_\mu \Box C^\mu \right]$$

$$\epsilon_{\mu\nu\rho\sigma\tau\pi} F^{\rho\sigma} F^{\tau\pi} + \dots$$

DIMENSIONAL REDUCTION

W. PAUL HOWE

- PROCEEDINGS 413
- NPB 181 ('81) 487

CAMBRIDGE 1980

3-LOOP	$N \leq 3$
N-LOOP	$N \geq 4$
(N-1)	NONLINEAR

n=1

- FERRARA-ZUMINO (4N) '78
- MOURA '01 (FULL OFF-SHELL)

SUPERSPACE !

$N = 8$ ON-SHELL SUPERSPACE SUPERGRAVITY

- L. BRINK, P. HOWE PL 88B ('79)

TANGENT SPACE: $SL(2,\mathbb{C}) \times G \quad : (G = SU(8))$

GLOBAL: $G' \ (= E_7)$

$$T_{\underline{\alpha}\,\underline{\beta}}{}^{\underline{\dot\gamma}} = 2\,\epsilon_{\alpha\beta}\,\bar\chi^{abc\dot\gamma}$$

$$\mathcal{L}^{(3)} = \kappa^4 W^2 \bar W^2 E$$

$2_{N \leq 3}$

$$\int d^4x\, d^{4N}\theta\, \mathcal{L}^{(N)}(z)$$

$$\Delta_1 \equiv C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$$

$$\mathcal{L}^{(N)}(z) = \kappa^{2(N-1)} \epsilon^{\alpha\beta} \epsilon^{\dot\alpha\dot\beta} \chi_{\alpha abc} \chi_{\beta def} \bar\chi_{\dot\alpha}^{abc} \bar\chi_{\dot\beta}^{def} E$$

LIN. $\kappa^{2(N-1)} \Delta_1 \Box^{(N-3)} \bar\Delta_1$

- R. KALLOSH LEB. PREP. '80 (3-LOOP LIN.)

STRING THEORY

- SUGR. EFFECTIVE THEORY
- D-BRANES, p-BRANES ; WV. THEORY
- M-THY → D=11 SUGR
- ⋮

$$\mathcal{L}_{DBI} + \mathcal{L}_{WZ}$$

- GREEN, HARVEY, MOORE '97
- CHEUNG, YIN '98

$$T_p \int C \wedge \mathrm{tr}\, e^{F} \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}$$

p : POINTRYAGIN CLASS

$$\sqrt{\hat{A}(R)} = 1 - \frac{1}{48} p_1(R) + \frac{1}{2560}\left(7 p_1^2(R) - \cdots\right.$$

$$(R_T)_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \delta_{\pi'\tau'}\left(\Omega^{\pi'}_{\mu[\rho}\,\Omega^{\tau'}_{\sigma]\nu}\right)$$

$$(R_N)_{\mu\nu}{}^{\pi'\tau'} = -R^{\pi'\tau'}{}_{\mu\nu} + g^{\rho\sigma}\left(\Omega^{\pi'}_{\rho[\mu}\,\Omega^{\tau'}_{\nu]\sigma}\right)$$

Ω 2ND FUNDAMENTAL FORM

$$\mathcal{L}_{DBI} = T_p \sqrt{-\det\left[(G_{\mu\nu}+B_{\mu\nu})\partial_\mu X^\mu \partial_\nu X^\nu + F_{\mu\nu}\right]}\; e^{-\phi}$$

- ∂F - TERMS $\mathcal{O}(\alpha'^2)$. ANDREEV, TSEYTLIN '88
- $\partial^2 X$ - TERMS $\mathcal{O}(\alpha'^2)$. BACHAS, BAIN, GREEN '99
  FOTOPOULOS '01

BOSONIC.

• WYLLARD '01

$$T_p\, e^{-\phi}\sqrt{\bar{g}}\left[1 - A(\alpha')^2\left\{(R_T)_{\mu\nu\rho\sigma}(R_T)^{\mu\nu\rho\sigma} - 2(R_T)_{\mu\nu}(R_T)^{\mu\nu} - (R_N)_{\mu\nu\rho'\sigma'}(R_N)^{\mu\nu\rho'\sigma'} + 2\bar{R}_{\rho'\sigma'}\bar{R}^{\rho'\sigma'}\right\}\right]$$

AMBIGUITY $\sim \Omega^{\lambda'}_{\mu}{}^{\mu}$.

Q: κ-SYMMETRIC BRANE ACTION W. "RIGIDITY" TERM ?

- "RIGID" STRING (BOS). · POLYAKOV '86
- (SPINNING) WEYL INV. · U.L., ROČEK, VAN NIEUWENHUIZEN '87 " " '88
- $\mathcal{N}$-SYM. (PART. RES.) · CURTRIGHT, VAN NIEUWINHUIZEN '87
- $\mathcal{N}$-SYM. SUPERPARTICLE · IVANOV, KAPUSTNIKOV '91
- · GAUNTLETT '91
- p-BRANE ?

$$S_{GS} = T\int d^{p+1}\xi \left[\sqrt{-\gamma} + \epsilon^{\mu\nu}\bar{\theta}\Gamma_\mu\partial_\nu\theta\right]$$

$$\Pi^{\underline{m}}_{\mu} \equiv \partial_\mu X^{\underline{m}} - i\bar{\theta}\Gamma^{\underline{m}}\partial_\mu\theta$$

$$\gamma_{\mu\nu} \equiv \Pi^{\underline{m}}_{\mu}\Pi^{\underline{n}}_{\nu}\eta_{\underline{m}\underline{n}}$$

$\mathcal{N}$-SYMMETRY

$$\delta\theta = \epsilon$$
$$\delta X^{\underline{m}} = i\bar{\epsilon}\Gamma^{\underline{m}}\theta$$

$$\delta\theta = \kappa$$
$$\delta X^{\underline{m}} = -i\bar{\kappa}\Gamma^{\underline{m}}\theta$$
$$\kappa = p\!\!\!/\,\kappa \;;\quad p^2 = p$$

POLYAKOV

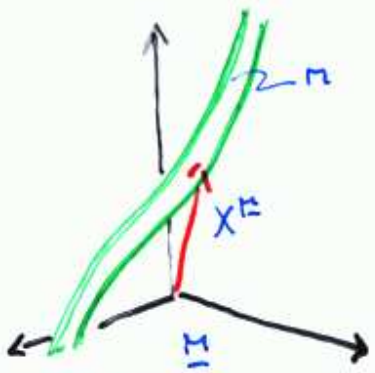
$$T\int d^2\xi\sqrt{-\gamma}\left[1+\mu\,\Omega^2\right]$$

BOSONIC

$$\Omega^{\sigma'}_{\mu}{}^{\underline{m}}\cdot\Omega^{\sigma'}_{\nu}{}^{\underline{n}} \sim \eta_{\underline{m}\underline{n}}\partial_\mu\delta X^{\underline{m}}\partial_\nu\delta X^{\underline{n}}$$

SUPEREMBEDDINGS

* SOROKIN, TKACH, VOLKOV ZHELTUKHIN '89



$X^{\underline{M}} : M \to \underline{M}$

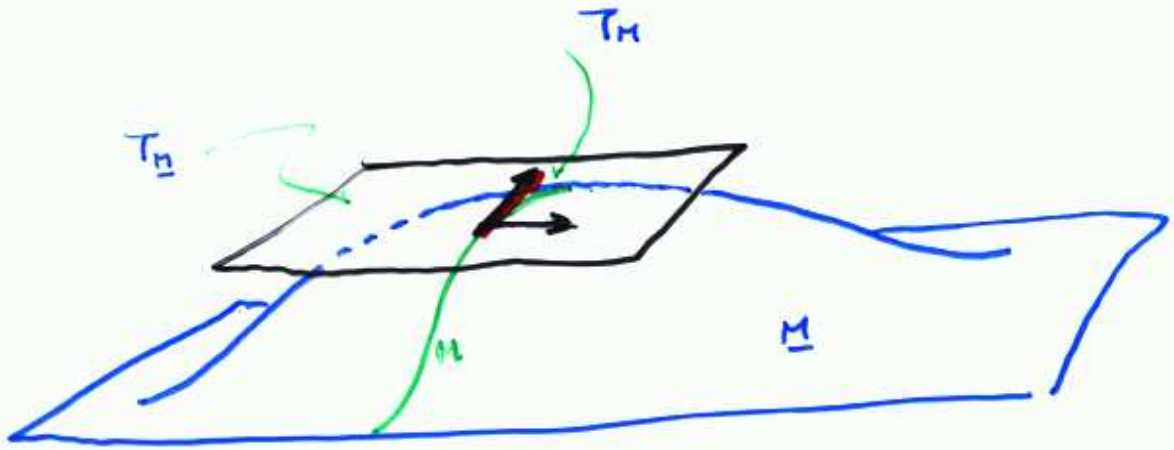
(BOTH SUPERSPACES)

E_M^A, $E_{\underline{M}}^{\underline{A}}$ ← TANGENT, ← WORLD

$\underline{A} = (\underline{a}, \underline{\alpha})$

$$E_A^{\underline{A}} \equiv E_A^M \partial_M X^{\underline{M}} E_{\underline{M}}^{\underline{A}}$$

THE EMBEDDING MATRIX



THE EMBEDDING CONDITION

$$E_{\alpha}^{\hat{\underline{a}}} = 0$$

$$T_M^F \subset T_{\underline{M}}^F$$

$$T_{\underline{MN}}{}^{\underline{A}} = 2\,\nabla_{[\underline{M}} E_{\underline{N}]}{}^{\underline{A}}$$

$$2\,\nabla_{[A} E_{B]}{}^{\underline{C}} + T_{AB}{}^{C} E_{C}{}^{\underline{C}} = (-)^{A(B+\underline{B})} E_{\underline{B}}{}^{\underline{B}} E_{\underline{A}}{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}}$$

- CHOOSE THE GEOMETRY OF $\underline{M}$
- PARAMETRIZE THE EMBEDDING CONDITION ETC.
- EXPLORE THE CONSEQUENCES OF THE TORSION REL'N.

THE SUPERMULTIPLET THAT DESCRIBES THE EMBEDDING WILL TYPICALLY BE ON-SHELL FOR LARGE NUMBER OF SUSY. FOR $N \leq 16$ IT CAN ALSO BE OFF-SHELL.

κ-SYMMETRY

$$\delta X^{\underline{M}} = v^{M} \partial_{M} X^{\underline{M}}$$

$$\delta X^{\underline{A}} \equiv \delta X^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} = v^{A} E_{A}{}^{\underline{A}}$$

THE EMB. MATRIX

$$v^{a} = 0 \;\Rightarrow\; \begin{cases} \delta X^{\underline{a}} = E_{\underline{a}}{}^{\underline{\alpha}} v^{\alpha} = 0 \\ \delta X^{\underline{\alpha}} = v^{\alpha} E_{\alpha}{}^{\underline{\alpha}} \equiv \kappa^{\underline{\alpha}} \end{cases}$$

$$\kappa^{\underline{\alpha}} = \kappa^{\underline{\beta}} P_{\underline{\beta}}{}^{\underline{\alpha}}$$

PROJ. ONTO $T_M^F \, CT_{\underline{M}}^F$

$$P_{\underline{\beta}}{}^{\underline{\alpha}} = (E^{-1})_{\underline{\alpha}}{}^{\gamma} E_{\gamma}{}^{\underline{\beta}}$$

MEMBRANE IN FLAT D=4 SUPERSPACE

$\exists$ OFF-SHELL MULTIPLET.

$$E_\alpha^{\underline{a}} = 0$$

$$E_\alpha^{\underline{\alpha}} = u_\alpha^{\underline{\alpha}} + i h \delta_\alpha^{\alpha'} u_{\alpha'}^{\underline{\alpha}}$$

$$u = \begin{pmatrix} u_\alpha^{\underline{\alpha}} \\ u_{\alpha'}^{\underline{\alpha}} \end{pmatrix} \Updownarrow \begin{pmatrix} u_a^{\underline{a}} \\ u_3^{\underline{a}} \end{pmatrix}$$

$$T_{\alpha\beta}^{\ c} = -i (\gamma^c)_{\alpha\beta}$$

$$E_a^{\underline{a}} = (1+h^2) u_a^{\underline{a}}$$

D=3 SUGRA

- BROWN, GATES '79

$$T_{\alpha\beta}^{\ \gamma} = 0, \quad T_{\alpha b}^{\ c} = 0$$

$$T_{ab}^{\ c} = 0, \quad T_{a\beta}^{\ \gamma} = i (\gamma_a)_\beta^{\ \gamma} S$$

$$X = du u'$$

$$E_a^{\underline{\alpha}} = \Lambda_a^{\ \alpha} u_\alpha^{\underline{\alpha}} + \psi_a^{\ \alpha'} u_{\alpha'}^{\underline{\alpha}}$$

$$X_{a,b3} = \tfrac{1}{2} X_{(a,b)3} = X_{ab3}$$

ACTIONS

WANT

$$T_2 \int d^3\xi \left[\sqrt{-g}\,(1+\mu\,\Omega^2) + \text{FERMIONS} \right]$$

• HOWE, RAETZEL, SEZGIN '98

$$\sqrt{-g}\left(\frac{1-h^2}{1+h^2}\right)\Big| - \frac{1}{6}\epsilon^{\mu\nu\rho} C_{\mu\nu\rho}\Big| \qquad (\kappa\text{-SYM.}).$$

? WZ

BY DIM. AN. . HOWE, U.L. '01 hep-th/011036

$$\boxed{\begin{aligned} L_{(1)} &\equiv -\frac{i}{2}\Lambda^{a\alpha}\Lambda_{a\alpha} \\ L_{(2)} &\equiv -\frac{i}{2}(\gamma^{ab})^{\alpha\beta}\Lambda_{a\alpha}\Lambda_{b\beta} \end{aligned}}$$

MULT. BY ARB. $f(h)$...

$$\boxed{\begin{aligned} -i\nabla^\alpha\nabla_\alpha L_{(1)} &= \frac{1}{2}X_{ab3}X^{ab3} + \frac{h^2(2+9h^2)}{2(1+3h^2)^2} - \frac{2(1+2h^2+3h^4)}{(1+h^2)^2}(\nabla h)^2 \\ -i\nabla^\alpha\nabla_\alpha L_{(2)} &= \frac{1}{2}X_{ab3}X^{ab3} - \left(\frac{1}{2}+\frac{h^2(2+9h^2)}{(1+3h^2)^2}\right)X_3^2 \\ &\quad + \frac{4h^2(2+3h^2)}{(1+h^2)^2}(\nabla h)^2 \end{aligned}}$$

$\delta h \Rightarrow h(X_{ab3})$ (DIFF. EQN) ....

REMARCABLY:

$$L^{\circ}_{\mid} + \mu\left(L^{(1)}_{\mid} + \frac{1}{2} L^{(2)}_{\mid}\right)$$

$$= \sqrt{-\det g}\left(\frac{1-h^2}{1+h^2} + \frac{\mu}{2}\hat{\Omega}^2 - \frac{4\mu}{3(1+h^2)^2}(\nabla h)^2\right)$$

THE TRACELESS PART
OF THE 2ND FUNDAMENTAL FORM

$$\phi = \tanh^{-1} h$$

$$\delta\phi \Rightarrow \nabla^2\phi - \frac{3}{4\mu}\sin 2\phi = 0$$

SINE-GORDON !

OUTLOOK

- D-BRANES, HIGHER ORDER IN α'
- MODIFY THE EMBEDDING CONDITION
- .