INVENTING SYMMETRIES ?

B. JULIA ENS PARIS FRANCE

$D \geq 6$ 1975 KALUZA SCHERK + SCHWARZ

$D = 11$ { 1977 / 1978-79 } $U = E_{n(n)}(\mathbb{R})$ CJS / CJ

$N_4 \leq 6$ 1980 ADE EHLERS, G+H, J → MT_2, $N_4 = 0$

$D = 3 \to 2$ 1981 $U_2 = U_3^{(1)}$, E_9 ... GEROCH J

$D = 1$ 1982 E_{10}, $D_8^{\wedge\wedge} = DE_{10}$ (BE_{10}?, CE_{10}? ...) J

$U_1 \overset{\sim}{\hookleftarrow} U_2$ 1997-10119 CREMMER-JU-POPE + J

$\langle U_1, U_2 \rangle$ GENERATE G BEYOND U-DUALITY

1998-06106 CJLP II , { CJLP III / MT_2

WEYL ($U_3^{\wedge\wedge}$) CONTROLS ~1D CHAOS

DAMOUR HENNEAUX NICOLAI + J 2001-03094

SUPERGRAVITY AT 25

STONY BROOK Dec 1, 2001

I/ FINDING A-ACTIONS

WESS-ZUMINO 73-74 PARIS

SCHERK + SCHWARZ 75 PRINCETON

→ PROPAGANDA ...

PRINCETON .. KALUZA-KLEIN +

FERMIONS $d \geq 6 : 5+1$? $4+2$? ...

DYONS ⇒ . BOSON ↔ FERMION

1975 . LES HOUCHES (DYON) 1975

1976 . HARVARD LUNCH

1979 . DIRAC (TRIESTE)

. WITTEN (PARIS)

. SPONTANEOUS COMPACTIFICATION

- Q_{EL} / MASS
- SCALAR ?
- MISSED = PSB

= BPS ≠ JZ !

LATER { DUALITY
CHIRALITY }

(76) 1977 (78)

GSO PROJECTION

$d=2$ SUSY$_1$ $\rightleftarrows$ $D=10$ SUSY$_{\substack{I \\ II}}$

* SCHERK + al.

SUPER YANGMILLS$_{10 \to 4}$

GELL-MANN $10-4 = 6 \to SO(6)$

NEED $SO(8)$ - $SU(8)$

TRIALITY.

NAHM $\parallel d$ SPECTRUM

$\hookrightarrow$ JOEL (PRIVATE)

RR $u^t \Gamma^i v$, $u^t \Gamma^{[ijk]} v$

II_A

A_μ $A_{\mu\nu\rho}$

SUPERGRAVITIES D = 4

N = 1 1976 FF vN (Z...)

ENS SUMMER INSTITUTE

N = 2

N = 3

N = 4 SCALARS : NON POLYNOMIAL

SO(4) ↔ SU(4)

DUALITIES

$dF \equiv 0$

$d{*}F \approx 0 \iff dG \equiv 0$ $\quad \boxed{G \approx {*}F}$

(Z.S.F.C.) $\rightarrow \dfrac{SU(1,1)}{U(1)}$ $\quad K_{ij} = \begin{pmatrix} -1 & \\ & +1 \end{pmatrix}$

N = 8 LOWEST ORDER DW + F

↓

D = 10 $\quad 8_B \times 4 = 2 \times 16$

D = 11 $\quad = 1 \times 32$

2 NEW TOOLS

AFTER B. ZUMINO'S PREPARATION

a) FIERZ (PAULI - F. de BROGLIE ...)

ψ^4

b) GUPTA PROGRAM (G-DESER - $F^2 \vee N$ - ... - CJS)

DEFORMATION OF

{GROUP, REPRESENTATION, INVARIANT}

∞ DIMENSIONAL

TYPE II SUPERSTRINGS. ...

THE N=4 EXTENDED THEORY (CHAMSEDDINE)

: 64 D.O.FREEDOM. (COMPLETED)

d	$g_{\mu\nu}, A_{\mu\nu}, \varphi$
10	$\mathbb{R}\,\vert_0^1$
9	$\mathbb{R}\times\mathbb{R}\,\vert_0^2$
8	$\frac{SO(2,2)}{SO(2)^2}\times\mathbb{R}\,\vert_2^7$
7	$\frac{SO(3,3)}{SO(3)^2}\times\mathbb{R}\,\vert_6^{16}$
6	$\frac{SO(4,4)}{SO(4)^2}\times\mathbb{R}\,\vert_{12}^{29}$
5	$\frac{SO(5,5)}{SO(5)^2}\times\mathbb{R}\,\vert_{20}^{46}$
4	$\frac{SO(6,6)\times SU(1,1)}{SO(6)^2\times U(1)}\,\vert_{32}^{69}$
3	$\frac{SO(8,8)}{SO(8)^2}\,\vert_{56}^{120}$
2	$O_8^{(1)}$
1	HYPERBOLIC?

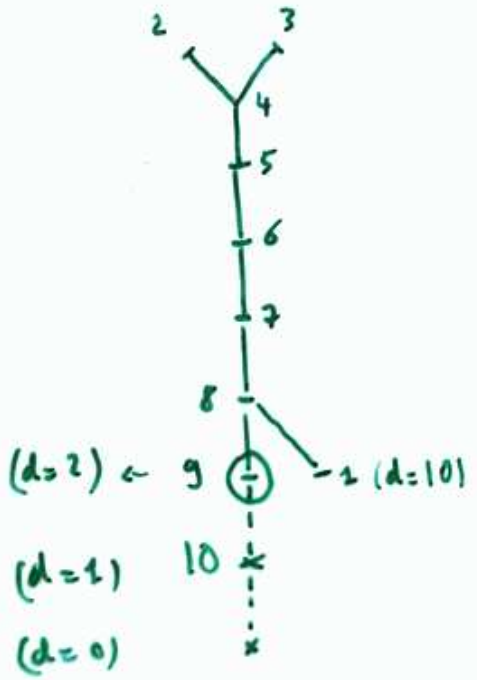
$2+d-N=7$

$\rightarrow$ $= SL(4)!\times\mathbb{R}$

BOURBAKI (456) p 134 $\leftarrow$

0

-1



E_{10}, D_{10} : THE ONLY 2 SH.SL.KM ALG.

II / INVENTIONS ?

- $U_{\mathbb{R}}$ RIGID = GLOBAL SYMMETRY

① NON-COMPACT ($N=4$ ZCFS)

$\left\{ \begin{array}{l} \text{NO GHOSTS} \\ \quad \text{KILLING}\big|_{KU^\perp} > 0 \end{array} \right.$ $N \geq 5$ ...

$(h, e, f) \equiv (\sigma_3, \sigma_+, \sigma_-)$ $\mathbb{R}$-GENERATE $SO(2,1)$

$ih, (e-f), i(e+f)$ $\mathbb{R}$-GENERATE $SO(3)$

↑ COMPACT CARTAN GENERATOR

$SO(8,24)$ $\left\{ \begin{array}{l} \text{PUZZLE} \\ 16+2=18>10 \end{array} \right.$

QUANTUM DUALITY IS "$U_{\mathbb{Z}}$" (W-S)

- KU = LOCAL SYMMETRY $\leftrightarrow$ LORENTZ

② $\cap$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\cap$

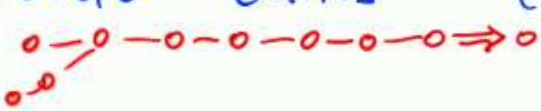
$\hat{U}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $GL(D, \mathbb{R})$

PERMITS FERMIONS

- AFFINE EXTENSION

③ $D=2$, $D\sim 1$ GENERATES MAGNETIC DUALS ...

- HYPERBOLIC CHAOS (NEGATIVE CURVATURE)

BE_{10}  $A_1^{\wedge\wedge}$

$10 = 10$

↑ MODULAR STRINGS

↖ GROUPS ↙ LIE ↙ KAC-MOODY AFFINE ↖ KAC-MOODY HYPERBOLIC

(BORCHERDS ?)

HETEROTIC, I. STRINGS

HAVE RANK 10 CHAOS

OBSERVED BY OBOUR + KENNEAUX 2000-12172

RELATED TO TITS-SATAKE THEORY

IN HANANY-KEURENTJES-J. 2001-12xxx

INVENTIONS !

(BEYOND DISCOVERIES)

④ INSTANTONS EVERYWHERE

(QUANTUM HOPES)

FINITE DIM. SUPERALGEBRA

$\Theta = \{T, \tilde{T}\}$ (EVEN)

$A = e^{A^a \Theta_a}$ ALL p-FORMS

$A \to A\, e^{\lambda^a \Theta_a}$ $(d\lambda = 0)$

UNIVERSAL NON-ABELIAN STRUCTURE (ON T^4)

$$dA\,A^{-1} \equiv \mathcal{F} \approx S(*\mathcal{F})$$

$S(T) = \tilde{T}$

$S(\tilde{T}) = (*)^2 T \equiv \pm T$

Ex : $e^{A_{(3)}\,\theta + B_{(6)}\,\tilde{\theta}}$ (ODD, EVEN)

$\{\theta\theta\}_+ = \kappa\,\tilde{\theta}$

NC $\mathbb{Z}$ (AND $\mathbb{Z}_2$) GRADED

IWASAWA $A = \sigma . d . t_r^+$

$SO(n) \backslash SL(n)$

HIDDEN SYMMETRY $\Leftrightarrow$ FIXED GAUGE

RECALL MAXWELL'S GAUGE INVARIANCE

NEXT 3 INVENTIONS

WE HAVE FOUND TRIANG. (G)

∪

TRIANG (U)

BOREL ($E_{4(+n)}$)

STILL MYSTERIOUS BECAUSE ≠ RIGID

SUPERALGEBRA = VERY SOLVABLE

i.e.

VERY NON-SIMPLE

BUT STILL NON-ABELIAN

$T_1 T_2 = T_3 \ldots$

1'

NEED → FULL G := RIGID?

$G \supset KG \not\in$ TRIANG (G)

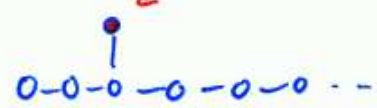
↓

PERMITS FERMIONS

BUT RECALL ALSO $A_{ijk} = \bar{u} \Gamma_{ijk} v$

SO(8) TRIALITY

E_n



A_{n-1}

(2') GRAVITY DOUBLING $d=4$ $S_* R = R$

- NUT CHARGE ARBITRARY Λ_{cosm} !

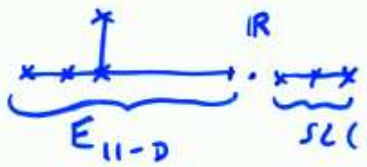
(1) - CONTRAST | YANG-MILLS - DUAL?
- DOUBLING

$G_{(7)} = dB_{(6)} + \kappa A_{(3)} F_{(4)}$

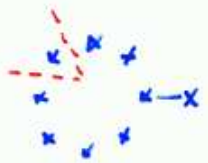
- BEYOND LIE GROUPS ...

- CONSERVATIVE ATTEMPTS

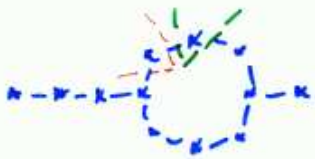
J. 1998 - 05083 CARGESE LECTURES



$\subset E_{11}$ FOR ANY D.



$A_7^{++} \equiv EA_9$ LAST CHAOS FOR PURE GRAVITY $D \leq 10$



F_{12} ?

$SL(11-D)$ FROM TORUS = 1980-81-82(J)

YET EVEN E_{10} REMAINS CONJECTURE

(3') FERMIONIZATION

MAJOR SPIN-OFF CLASSICAL LIMITS ↔ DUALITIES

FERMIONS, DYONS AND D-BRANES ARE $\frac{1}{2}$ WAY.

FERMION - BOSON DEMOCRACY.

MAGIC TRIANGLES

1) FROM PURE 4D. SUGRAS

$m=0$ (NO MATTER)

$D-3 \leftrightarrow 7-N_4$

$$\boxed{4D-12 \leftrightarrow 28-\#Q_{SUGRA}}$$

2) FROM OXYDISINTEGRATION OF $KE_n \backslash E_n(n)$

$CJLP_{III}$

del Pezzo's SPECIAL ALGEBRAIC SURFACES

+

MORE ...

$D = 11$	-								
$D = 10$	$\mathbb{R}$								
$D = 9$	$\mathbb{R} \times A_1$	$\mathbb{R}$							
$D = 8$	$A_1 \times A_2$	$\mathbb{R} \times A_1$	A_1						
$D = 7$	E_4	$\mathbb{R} \times A_2$	$\mathbb{R} \times A_1$	$\mathbb{R}$	-				
$D = 6$	E_5	$A_1 \times A_3$	$\mathbb{R} \times A_1^2$	$\mathbb{R}^2$	$\mathbb{R}$				
$D = 5$	E_6	A_5	A_2^2	$\mathbb{R} \times A_1^2$	$\mathbb{R} \times A_1$	A_1			
$D = 4$	E_7	D_6	A_5	$A_1 \times A_3$	$\mathbb{R} \times A_2$	$\mathbb{R} \times A_1$	$\mathbb{R}$		
$D = 3$	E_8	E_7	E_6	E_5	E_4	$A_1 \times A_2$	$\mathbb{R} \times A_1$	$\mathbb{R}$	-

Table 3: Disintegration (*i.e.* Oxidation) for E_n Cosets

"SPLIT" FORMS

$E_n(n)$