Joining the Clash: de Sitter vs. SUSY

(Ferrara, 1977)

Ferrara's Observation in form of 4D, N = 1 Supergravity Torsions and Curvatures

$$
\begin{aligned}
[\, \nabla_\alpha \, , \; \nabla_\beta \, \} &= \; - \, 2 \, \lambda \, \mathcal{M}_{\alpha\beta} \quad , \\
[\, \nabla_\alpha \, , \; \overline{\nabla}_{\dot\alpha} \, \} &= \; i \nabla_{\underline{a}} \quad , \\
[\, \nabla_\alpha \, , \; \nabla_{\underline{b}} \, \} &= \; - \, \lambda \, C_{\alpha\beta} \, \overline{\nabla}_{\dot\beta} \quad , \\
[\, \nabla_{\underline{a}} \, , \; \nabla_{\underline{b}} \, \} &= \; 2 \, \lambda\overline{\lambda} \, (\, C_{\dot\alpha\dot\beta} \, \mathcal{M}_{\alpha\beta} \; + \; C_{\alpha\beta} \, \overline{\mathcal{M}}_{\dot\alpha\dot\beta} \,) \quad ,
\end{aligned}
$$

Note the sign of the last term is independent of how λ is chosen. In the conventions of *Superspace* this is characteristic of an AdS geometry.

A de Sitter (dS) geometry requires

$$
[\, \nabla_{\underline{a}} \, , \; \nabla_{\underline{b}} \, \} = \; - \, 2 \, \lambda\overline{\lambda} \, (\, C_{\dot\alpha\dot\beta} \, \mathcal{M}_{\alpha\beta} \; + \; C_{\alpha\beta} \, \overline{\mathcal{M}}_{\dot\alpha\dot\beta} \,) \quad ,
$$

with the obvious difference of a sign.

For a long time this apparent theorem was supported by explicit constructions.

(a.) Gauged 4D SO(2), SO(3) SUGRA Actions

- (i.) Freedman & Das (1977)
- (ii.) Das (1977)

(b.) Gauged 4D SO(4) SUGRA Action

- (i.) Das, Fischler & Roček (1977)

In all of these models the sign of the cosmological constant agreed with the general argument of Ferrara.

The situation became even murkier when a second version of 4D, N = 4 SG was found.

(a.) 4D SU(4) SUGRA Action

- (i.) Cremmer, Scherk and Ferrara (1977)

Conformal Map Duality

Cremmer-Scherk-Ferrara

Let two complex variables $\mathcal{W}$ and $\mathcal{Z}$ be defined by

$$\mathcal{W} \equiv A(x) + iB(x) \quad ,$$
$$\mathcal{Z} \equiv A'(x) + iB'(x) \quad .$$

and a conformal mapping is defined by

$$\mathcal{Z} \to \frac{\mathcal{W}}{\mathcal{W} - 1}$$

and induces the transformation

$$\frac{1}{1 - |\mathcal{Z}|^2} |\partial \mathcal{Z}|^2 \to \quad ,$$
$$\frac{1}{1 - \mathcal{W} - \overline{\mathcal{W}}} |\partial \mathcal{W}|^2 \quad .$$

and a conformal mapping is defined by

$$\mathcal{W} = \frac{\mathcal{Z}}{\mathcal{Z} - 1}$$

4D, SO(4) SG $\longleftrightarrow$ 4D, SU(4) SG

When the SU(2) $\otimes$ SU(2) sub-group (4D, N = 4 SG has only six spin-one fields) was gauged,

(a.) Gauged 4D SU(2) $\otimes$ SU(2) SUGRA Action
Freedman & Schwarz (1978)

it was found that this action also agreed with the Ferrara observation.

Finally, one additional formulation of 4D, N = 4 SG was found

(a.) Anomaly-Free 4D SU(4) SUGRA Action
Nicolai & Townsend (1981)

This action is connected by Hodge duality on the B field (i.e. replacement by a 2-form) in the SU(4) model.

Breakthrough: De Sitter Space As Spontaneously Broken SUSY Phase

In 1982, I began to study the N = 4 supergeometry and discovered that from this perspective all component level formulation were derivable within a universal setting.

$$
\begin{aligned}
\nabla_{\alpha i} &= \mathrm{E}_{\alpha i}{}^{\underline{M}} D_{\underline{M}} + \omega_{\alpha i\, \underline{c}}{}^{\underline{d}} \mathcal{M}_{\underline{d}}{}^{\underline{c}} + \Gamma_{\alpha\, i}{}^{kl}\, \mathcal{T}_{kl} \\
\nabla_{\underline{a}} &= \mathrm{E}_{\underline{a}}{}^{\underline{M}} D_{\underline{M}} + \omega_{\underline{a}\, \underline{c}}{}^{\underline{d}} \mathcal{M}_{\underline{d}}{}^{\underline{c}} + \Gamma_{\underline{a}\, i}{}^{kl}\, \mathcal{T}_{kl} \\
[\, \nabla_{\alpha i} \, , \, \nabla_{\beta j} \,\} &= \ \dots \ + \ C_{\alpha\beta}\, \mathcal{E}_{ij}{}^{kl}\, \mathcal{T}_{kl} \quad ,
\end{aligned}
$$

The quantity $\mathcal{E}_{ij}{}^{kl}$ was required to be an invertible 6 x 6 matrix and given the spectrum of component fields had be be of the form (Gates, 1983)

$$
\mathcal{E}_{ij}{}^{kl} \ = \ \delta_i{}^{[k}\, \delta_j{}^{l]}\, U(W) \ + \ \epsilon_{ij}{}^{kl}\, V(W) \quad ,
$$

and the superspace Bianchi identities implied the “modulus choices”

$$
|U|^2 \ - \ |V|^2 = \begin{pmatrix} 1 \\ 0 \\ -\,1 \end{pmatrix} \quad ,
$$

The uniqueness of this last result was derived in a later work (Gates & Zwiebach, 1984). The quantity $\mathcal{T}_{kl}$ denote the generators of a rank six gauge group. Under these results all Bianchi identities were found to be satisfied. The quantity W is a chiral superfields whose leading components appear in the Cremmer-Scherk-Ferrara conformal map.

From the point of view of supergeometry, (almost) the only freedom that one has in describing 4D, N = 4 SG is in the "modulus choice" and the rank six gauge group choice. Some (but not exhaustive) choices are; $\mathcal{Z}^{(6)}$, $SO(4)$ and $SU(2)\otimes SU(2)$ and $\mathcal{Z}^{(3)}\otimes SU(2)$.

	$+1$	0	-1
$\mathcal{Z}^{(6)}$	FD, D, DFR	CSF	X
$SO(4)$	DFR	$--$	X
$SU(2)\otimes SU(2)$	GZ	FS	X
$\mathcal{Z}^{(3)}\otimes SU(2)$	X	GV	X

FD $\equiv$ Freedman & Das , D $\equiv$ Das , DFR $\equiv$ Das, Fischler & Roček , CSF $\equiv$ Cremmer, Scherk & Ferrara, GZ $\equiv$ Gates & Zwiebach , GV $\equiv$ Gubser & Volkov (2000).

The modulus choice = +1 and gauge group = $SU(2)\otimes SU(2)$ model was the first extended SUGRA model where it was shown that de Sitter space can occur as a spontaneously broken phase of an extended SUGRA model.

For example, prior to this construction, the gauged SO(8) model of de Wit and Nicolai was shown not to admit de Sitter backgrounds in a spontaneously broken phase.

Nicolai-Townsend Theory

It was later shown (Gates & Durachta) that this version of N = 4 SUGRA exist in a superspace formulation where the complex chiral superfield W is replaced by a real scalar superfield $\mathcal{V}$ and in addition to the superspace supercovariant derivative, it is necessary to introduce a super 2-form superfield for a completely geometrical description. This exhaust all freedom in superspace to describe the N = 4 SUGRA models.

The De Sitter model acted to spur the discovery of more such models first byHull, Hull & Warner and others to the case of the $N = 8$ models.

1) "A New Gauging of N=8 Supergravity," C. M. Hull, Phys. Rev. D30 (1984) 760.

2) "Noncompact Gaugings of N=8 Supergravity," C. M. Hull, Phys. Lett. B142 (1984) 39.

3) "More Gaugings of N=8 Supergravity," C. M. Hull, Phys. Lett. B148 (1984) 297.

4) "The Minimal Couplings & Scalar Potentials of the Gauged N=8 Supergravities," C. M. Hull, Class. Quant. Grav.2 (1985) 343.

5) "The Structure of the Gauged N=8 Supergravity Theories," C. M. Hull and N. P. Warner, Nucl. Phys. B253 (1985) 650.

6) "The Potentials of the Gauged N=8 Supergravity Theories," C. M. Hull and N. P. Warner, Nucl. Phys. B253 (1985) 675.

Some Loose Ends About De Sitter vs. SUSY in SUGRA Models

It is perhaps useful to point out that there are two tantalizing hints that the issue of de Sitter backgrounds in SUGRA models may yet still hold some more surprises.

(Hint A.)

One of these hints was actually first noted in 1983 in the book *Superspace* (hep-th/0108200) on page 336, where it is observed that the choice of auxiliary fields, required of an off-shell SUGRA multiplet, is sensitive to the presence of backgrounds spaces of constant curvature.

The point was made that the supergeometry of the Breitenlohner auxiliary fields can be cast in the form

$$
\begin{aligned}
[\,\nabla_\alpha \, , \, \nabla_\beta \,\} &= \tfrac{1}{2} T_{(\alpha} \nabla_{\beta)} \; - \; 2\,(\overline{R} \, + \, T^\alpha T_\alpha) \mathcal{M}_{\alpha\beta} \quad , \\
[\,\nabla_\alpha \, , \, \overline{\nabla}_{\dot\beta} \,\} &= i \, \nabla_{\underline{a}} \; - \; \tfrac{1}{2} (\, T_\alpha \, \overline{\nabla}_{\dot\beta} \, + \, \overline{T}_{\dot\alpha} \, \nabla_\beta \,) \quad , \\
[\,\nabla_\alpha \, , \, \nabla_{\underline{b}} \,\} &= \; \cdots \quad , \\
[\,\nabla_{\underline{a}} \, , \, \nabla_{\underline{b}} \,\} &= \; \cdots \quad .
\end{aligned}
$$

Together with the condition

$$\overline{R} \ = \ - \tfrac{1}{2} \nabla^{\alpha} T_{\alpha} \qquad ,$$

and it turns out that there is no globally supersymmetric limit in which these results return to the form first implied by Ferrara's work.

This phenomenon was investigated in term of the compensating field formalism (Deo-Gates, 1984) and that study also supported this assertion.

This may not be an academic matter, especially if the model suggested by Kallosh (hep-th/0110271) is thought to provide a way in which to reconcile quintessence and SUSY in a stringy context. *All* known N = 2 off-shell SUGRA multiplets possess a subsector N = 1 off-shell SUGRA multiplet whose auxiliary field *are* the Breitenlohner set.

One final point to note about this phenomenon is that it may well provide an example of a model in which the breaking of flavor symmetry results in the breaking of SUSY. The point is that usually in breaking internal flavor symmetries most such models generate a cosmological term. But such a term for this SUGRA theory must necessarily drive SUSY breaking so that the two would be intimately linked.

(Hint B.)

The works of de Wit and Nicolai and as well Hull et. al. both begin at the starting point of 4D, N = 8 SUGRA where all of the 70 spin-0 fields are represented by scalars. If there were no other options, then one might not raise the issue of additional presently unknown gauged 4D, N = 8 SUGRA models.

There is another option, toroidal compactification of type-II supergravity theories are such an option. In particular the straightforward construction from this point does not lead to a theory possessing 70 scalars nor to a model possessing SU(8) symmetry.

In fact, the reduced theory without using dualities (as was done by Cremmer and Julia in their original work) possess some 2-forms in place of scalars.

In particular the group Spin(6) seems to play the role of organizing the N = 8 SUGRA fields into its representations.

$\mathbf{D = 10}$, $\mathbf{N = 2A}$ Supergravity Reduction

D = 10	D = 4
$\hat{e}_{\hat{\mu}}{}^{\hat{m}}$	$e_{\underline{m}}{}^{\underline{a}},\ \varphi_{(\hat{\alpha}\hat{\beta})},\ \varphi,\ \tilde{A}_{\underline{m}}{}^{\hat{\alpha}},\quad (\varphi_{\hat{\alpha}}{}^{\hat{\alpha}} = 2\varphi)$
$\hat{\psi}_{\hat{m}}$	$\psi_{\underline{m}},\ \psi^{\hat{\alpha}},\ \psi,\quad (\Gamma^{\hat{\alpha}}\psi_{\hat{\alpha}} = 0)$
$\hat{\chi}$	χ
$\hat{A}_{\tilde{\mu}}$	$\tilde{A}_{\underline{m}},\ \varphi_{\hat{\alpha}}$
$\hat{B}_{\hat{\mu}\hat{\nu}}$	$B_{\underline{mn}},\ A_{\underline{m}\hat{\alpha}},\ \varphi_{[\hat{\alpha}\hat{\beta}]}$
$\hat{A}_{\hat{\mu}\hat{\nu}\hat{\rho}}$	$B_{\underline{mn}\hat{\alpha}},\ A_{\underline{m}\hat{\alpha}\hat{\beta}},\ \varphi_{\hat{\alpha}\hat{\beta}\hat{\gamma}}$
$\hat{\phi}$	ϕ

In particular, there are two points of note.

(a.) The eight gravitini in the theory may be regarded as forming the spinor representation of Spin(6) and

(b.) the spin-one fields (whose superspace field strength extensions largely determine the spin-0 content) are in multiplicities of 1 + 6 + 6 + 15 which is exactly what is needed to for the gauge group

$$SO(2) \otimes SO(4) \otimes SO(4) \otimes SO(6)$$

D = 10, N = 2B Supergravity Reduction

D = 10	D = 4
$e_{\underline{a}}{}^{m}$	$\begin{pmatrix} \hat{e}_{\underline{a}}{}^{m} & A_{\underline{a}}{}^{\hat{m}} \\ 0 & \Delta_{\hat{a}}{}^{\hat{m}} \end{pmatrix}$
$G(B)_{\underline{abc}}$	$G(B)_{\underline{abc}},\ G(B)_{\underline{ab}\hat{c}},\ G(B)_{\underline{a}\hat{b}\hat{c}}$
Φ	Φ
A	A
$F(A)_{\underline{abc}}$	$F(A)_{\underline{abc}},\ F(A)_{\underline{ab}\hat{c}},\ F(A)_{\underline{a}\hat{b}\hat{c}}$
$F(A)_{\underline{abcde}}$	$F(A)_{\underline{ab}\hat{c}\hat{d}\hat{e}},\ F(A)_{\underline{a}\hat{b}\hat{c}\hat{d}\hat{e}}$

Similar to the last case, there are two points of note.

(a.) The eight gravitini in the theory may be regarded as forming the spinor representation of Spin(6) and

(b.) the spin-one fields (whose superspace field strength extensions largely determine the spin-0 content) are in multiplicities of 6 + 6 + 6 + 10.

The question of whether a successfully gauged version of either the N = 8A theory or N = 8B theory and which leads to models that are distinct from those already elucidated by Hull's efforts seems to be a worthwhile task.

Joining the Clash: De Sitter vs. SUSY in Stringy σ-Models

The challenge of describing how de Sitter background geometries occur in the context of extended supergravity models was overcome models by showing that such constructions exist. However, it cannot be over emphasized that in the de Sitter phase, all supersymmetries are spontaneously broken.

Some years ago, it was shown (Gates & Siegel) how to write world sheet NSR σ-models which describe all the 4D, N = 4 massless modes of the heterotic string and wherein all bosonic condensates are explicitly represented in a (1,0) world sheet action.

$$
\begin{aligned}
S_{cond} \;=\; \frac{1}{2\pi\alpha'}\int d^2\sigma d\zeta\; E^{-1}\{\, & [i\tfrac{1}{2}g_{\underline{mn}}(X)\;(\nabla_{+}\mathbf{X}^{\underline{m}})(\nabla_{=}\mathbf{X}^{\underline{n}}) \\
& + i\tfrac{1}{2}b_{\underline{mn}}(X)\;(\nabla_{+}\mathbf{X}^{\underline{m}})(\nabla_{=}\mathbf{X}^{\underline{n}}) \\
& + \Phi(X)\Sigma^{+}\,] - \tfrac{1}{2}\eta_{-}{}^{\hat{I}}\nabla_{+}\eta_{-}{}^{\hat{I}} \\
& + i\tfrac{1}{2}[\; L_{+}{}^{\hat{\alpha}}(L_{=}{}^{\hat{\alpha}} + 2l_{=}{}^{\hat{\alpha}}) \;+\; \Lambda_{+}{}^{=}\tilde{L}_{=}{}^{\hat{\alpha}}\tilde{L}_{=}{}^{\hat{\alpha}}\;] \\
& + \frac{1}{2\sqrt{(2\pi\alpha')}}(\nabla_{+}\mathbf{X}^{\underline{m}})\eta_{-}{}^{\hat{I}}A_{\underline{m}\hat{I}\hat{J}}(X)\eta_{-}{}^{\hat{J}}\;]\;\} \quad ,
\end{aligned}
$$

where

$$
\begin{aligned}
& L_{+}{}^{\hat{\alpha}} \equiv \nabla_{+}\Phi_{L}{}^{\hat{\alpha}} \quad , \quad L_{=}{}^{\hat{\alpha}} \equiv \nabla_{=}\Phi_{L}{}^{\hat{\alpha}} \quad , \quad \tilde{L}_{=}{}^{\hat{\alpha}} \equiv L_{=}{}^{\hat{\alpha}} + l_{=}{}^{\hat{\alpha}} \quad , \\
& l_{=}{}^{\hat{\alpha}} \equiv \frac{1}{\sqrt{(2\pi\alpha')}}(\nabla_{=}\mathbf{X}^{\underline{m}})A_{\underline{m}}{}^{\hat{\alpha}}(X) + i\eta_{-}{}^{\hat{I}}\eta_{-}{}^{\hat{J}}\Phi_{\hat{I}\hat{J}}{}^{\hat{\alpha}}(X) \quad .
\end{aligned}
$$

$$
4D, N = 4 \text{ Sugra } - \;(g_{\underline{mn}},\, b_{\underline{mn}},\, \Phi,\; A_{\underline{m}}{}^{\hat{\alpha}})
$$

$$
4D, N = 4 \text{ YM } - \;(A_{\underline{m}\hat{I}\hat{J}},\, \Phi_{\hat{I}\hat{J}}{}^{\hat{\alpha}})
$$