SUPERGRAVITY

LIVES

AdS/CFT from a D3/D5 Brane System

0111135 with DeWolfe, Ooguri

Develop gravity + field thy sides of duality proposed by Karch + Randall 0105132

System of D3/D5's in IIB Str Thy:

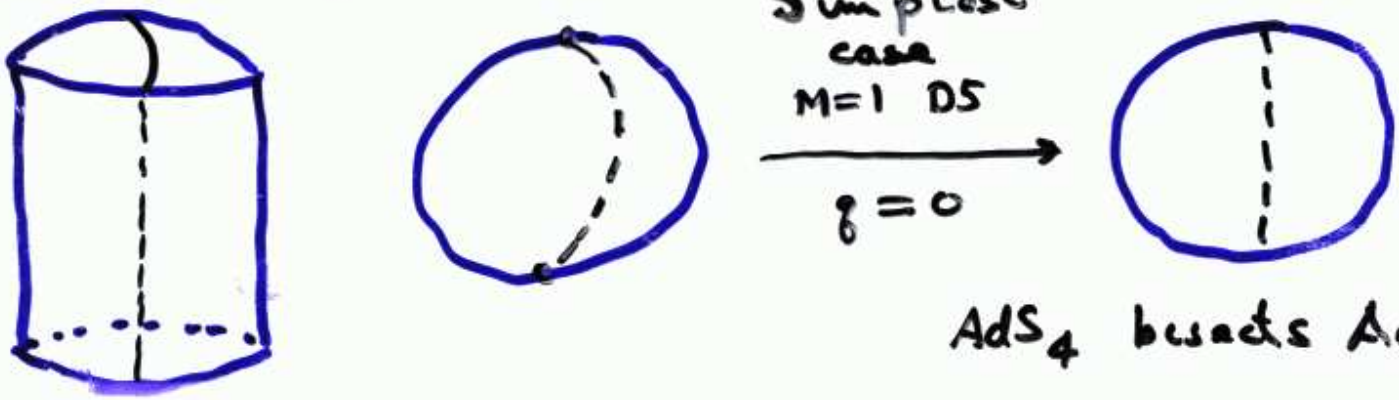
i) N D3's → usual $AdS_5 \times S_5$ geometry in near horizon limit

ii) stable position of D5 probe

D5 wraps max. radius $S_2 \subset S_5$

lives on $AdS_4 \subset AdS_5$

Basic setup in 5 dim:



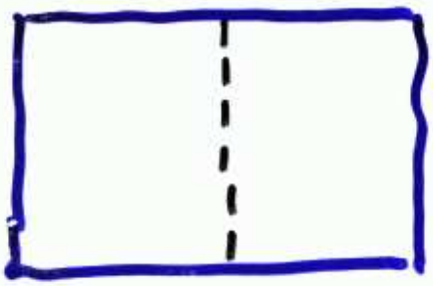
Two dualities in NH limit

1) Maldacena

IIB SG bulk fields in $AdS_5 \times S_5$ $\leftrightarrow$ $\mathcal{N}=4$ SYM on ∂AdS_5

2) D5 vector mult 5 5 open strs on $AdS_4 \times S_2$ $\leftrightarrow$ field thy on ∂AdS_4 elem fields are 3-5 strs → hypermultiplet

Field thy.



full $\mathbb{R}^4$ with usual $\mathcal{N}=4$ SYM

$\mathbb{R}^3$ defect with hyper.

Some $\mathcal{N}=4$ SYM fields interact on defect.

Bulk isom. gp. $SO(3,2) \subset SO(4,2)$ ← preserves AdS_4 brane

should match conf. gp of $Mink_4$ with $Mink_3$ defect, also $SO(3,2)$

Cardy

New type of dCFT = defect Conf Fld Thy

8 supercharges – $SU(2)_H \times SU(2)_V$ R-sym

$U(1)$ flavor sym.

Motivation:

1) no applics to particle thy in d=4 foreseen

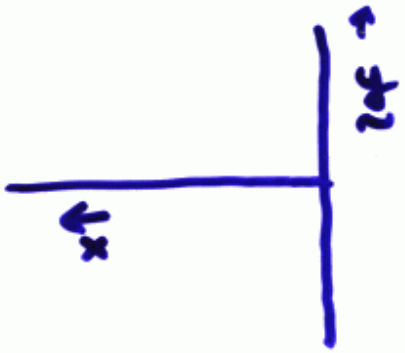
2) One of the few things you can do to a CFT_2 in pwt in a bdy, so it is useful to see if AdS/CFT extends to this!

S. Shenker 2001

3) Novel

Implications of conf. sym. for CFT's with bdy.

Cardy
McAvity-Osborn



$\mathcal{O}_4(x,\underset{\sim}{y})$ of scale dim Δ_4 on ambient R_4

$\mathcal{O}_3(\underset{\sim}{y}')$ of scale dim Δ_3 on R_3 defect

New Features:

i) non-vanishing $\langle \mathcal{O}_4(x,\underset{\sim}{y})\rangle = c/x^{\Delta_4}$

ii) $\langle \mathcal{O}_4(x,\underset{\sim}{y})\,\mathcal{O}_3(\underset{\sim}{y}')\rangle = \dfrac{c'}{x^{\Delta_4-\Delta_3}\left[x^2+(\underset{\sim}{y}-\underset{\sim}{y}')^2\right]^{\Delta_3}}$

iii) $\langle \mathcal{O}_4(x,\underset{\sim}{y})\,\mathcal{O}_4'(x',\underset{\sim}{y}')\rangle$

$$= \frac{1}{x^{\Delta_4}\,x'^{\Delta_4'}}\, f(\xi) \qquad \xi = \frac{(x_{\mu}-x'_{\mu})^2}{4xx'}$$

$\neq$ scale dims. arb. fn.

iv) normal CFT_3 behavior of $\langle \mathcal{O}_3\,\mathcal{O}_3 \ldots \mathcal{O}_3\rangle$

Hope to calculate these for $N\to\infty$, $g_s N \gg 1$ from duality.

III. Geometry

D3-D5 conventions a là Hanany-Witten

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	x	x	x				x			
D5	o	o	o	o	•	•	$x^6=0$	·	·	-

$SU(2)_H$ (wrapped) $\quad SU(2)_V$

$AdS_5 \times S_5$ metric from N D3's:

$$d_{10}s^2 = ds^2_{AdS_5} + ds^2_{S_5}$$

$$ds^2_{AdS_5} = L^2\left[\frac{dv^2}{v^2} + v^2(d\underset{\sim}{y}^2 + dx^2)\right]$$

$\underset{\sim}{y} = x^{0,1,2}$, $x = x^6$

AdS_4 at $x=0$

Be-spherical coords for round S_5

$$ds^2_{S_5} = L^2\left[d\psi^2 + \cos^2\psi\,(d\theta^2 + \sin^2\theta\, d\Omega^2) + \sin^2\psi\,(d\gamma^2 + \sin^2\gamma\, d\alpha^2)\right]$$

S_{2H} $\qquad$ S_{2V}

wrapped S_2 at $\psi = 0$

Its $\perp$ space in S_5 is

$$ds^2_\perp = L^2\left(d\psi^2 + \sin^2\psi\,(d\gamma^2 + \sin^2\gamma\, d\alpha^2)\right)$$

$$\approx dx_7^2 + dx_8^2 + dx_9^2$$

IV. Actions (very schematic)

$$S_{Tot} = S_{IIB} + S_{D5}$$

$$S_{D5} = S_{BI} + S_{WZ}$$

$$T_{D5} \sim \frac{1}{g_s \alpha'^3}$$

$$S_{BI} = -T_{D5} \int d^6\xi \, e^{\Phi/2} \sqrt{-\det\left(g^{PB}_{ab} + e^{-\Phi} \mathcal{F}_{ab} \right)}$$

$$S_{WZ} = -T_{D5} \int C_1 \wedge \mathcal{F}$$

KR result: the $AdS_4 \times S_2$ config of the D5 extremizes S_{D5} (with $g=0$ flux $\wedge \mathcal{F}_{ab}$)

2. Qual. form of S_{DS} for fluctuations

generic open str fields ψ (eg. B_a, X^i, ...)

" closed " " ϕ (e.g. $h_{\mu\nu}$, Φ, B_{ab})

We scale ϕ to normalize $\langle O_\phi O_\phi \rangle \sim 1$

" ψ " " $\langle O_\psi O_\psi \rangle \sim 1$

$$S_{DS} \to \int d^6 x \left[(\partial\psi)^2 + \frac{1}{N^{m+\frac{n}{2}-1} \lambda^{n/4-1/2}} \phi^m \psi^n \right]$$

N, λ dependence of correlators can be read directly from this.

Compare $N \to \infty$ $\lambda \gg 1$ gravity with planar graphs in QFT.

$$\langle \ldots \rangle = N^a f(\lambda) \xrightarrow[\lambda \ll 1]{} N^\alpha \lambda^\beta$$

Generically λ dependence disagrees,

$\Rightarrow$ no generic non-renorm thms

But N-dependence generically agrees.

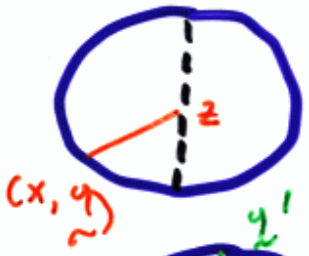
weak test that AdS/CFT can work

#3? $S_{BI} + S_{WZ}$ contain (after dim red. on S^5) &

$S_1 = \int d^4x \sqrt{g_{AdS_4}}\, \varphi$ and $S_2 = \int d^4x \sqrt{g}_{AdS_4} (\varphi\psi + \varphi^2)$

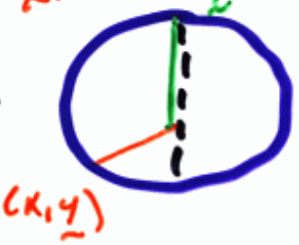
example φ = dilaton or h^a_a from expansion of $\sqrt{g_{10}}$

These lead to Witten diags. for "special correlators"

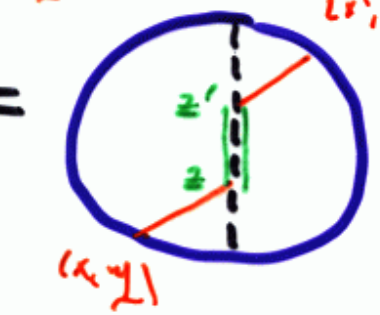
$\langle O_4 \rangle =$ 

bulk to bdy props.

$K_{\Delta_4} = C \left(\frac{z_0}{z_0^2 + x^2 + y^2} \right)^{\Delta_4}$

$\langle O_4 O_3 \rangle =$ 

$K_{\Delta_3} = C' \left(\frac{z_0}{z_0^2 + y'^2} \right)^{\Delta_3}$

$\langle O_4 O_4 \rangle =$ 

← bulk to bulk

$= G_3(z, z')$

AdS_4 integrals can be done by methods of MIT 9804058

$\langle O_4 \rangle = C_4 \lambda^{1/2} \frac{1}{x^{\Delta_4}} \Gamma\left(\frac{\Delta_4 - 3}{2}\right) \cdot \begin{pmatrix} \text{other } \Gamma\text{'s} \\ \text{with no} \\ \text{poles} \end{pmatrix}$

$\langle O_4 O_3 \rangle = C_{43} \frac{\lambda^{1/2}}{x^{\Delta_4 - \Delta_3} (x^2 + (y - y')^2)^{\Delta_3}} \Gamma\left(\frac{\Delta_4 - \Delta_3}{2}\right) \begin{pmatrix} \text{other} \\ \Gamma\text{'s} \end{pmatrix}$

$\langle O_4 O_4 \rangle$ can be done by method of "without really trying" 9905049

Features :

1. Spacetime forms req'd by conf symm of dCFT do emerge from gravity calcs.

Reason: usual AdS/CFT match of bulk isom gp + bdy conf gp.

2 Poles eg. at $\Delta_3 - \Delta_4 = 2n$ from generic couplings are not compatible with

$SO(6) \longrightarrow SU(2)_H \times SU(2)_V$ R sym

in actual 8 superdg. model. So couplings $C_{43} = 0$ in all but $\Delta_3 = \Delta_4$ case.

Similar to poles of $\langle \mathrm{Tr}\, X^k \mathrm{Tr}\, X^l \mathrm{Tr}\, X^m \rangle$

all but one susy case vanishes by $SO(6)$

one extremal case $k = l + m$ where

$0 \cdot \infty$ gives non-susy $\langle \quad \rangle$.

MIT '98
Princeton '98

V. KK reduction of D5 fields on $AdS_4 \times S_2$

$X^i(v, \underset{\sim}{y}, \theta, \varphi)$ $\quad i = 6, 7, 8, 9$

$B_a(v, \underset{\sim}{y}, \theta, \varphi)$ $\quad a = 0, 1, 2, 3, 4, 5$

A kindergarden KK problem:

i) no gravity

ii) high school harmonics in

$$X^i = \sum X^i_{\ell m}(v, \underset{\sim}{y})\, Y_\ell^m(\theta, \varphi)$$

$$B_a = \sum B_{\ell m}(v, \underset{\sim}{y})\, \varepsilon_{\alpha\beta} D^\beta Y_\ell^m(\theta, \varphi)$$

$\alpha, \beta \in S^2$

iii) non-trivial mixing of X^6 and B_α from S_{WZ}

Translate $m_\ell^2 \to \Delta_\ell$ for operators in CFT_3 by

$$\Delta_\ell = \tfrac{1}{2}\left(3 + \sqrt{9+4m_\ell^2}\right)$$

Full Results on KK decomposition:

Mode	Δ	$SU(2)_H$	$SU(2)_V$	
$(B_\ell + Z^c)^{(-)}$	ℓ	$\ell \geq 1$	0	← Primaries of short irrep of $OSp(4,4)$
Z^i	$\ell+2$	$\ell \geq 0$	1	
$(B_\ell + Z^c)^{(+)}$	$\ell+4$	$\ell \geq 0$	0	
B_M	$\ell+2$	$\ell \geq 0$	0	← lowest vector with $\Delta = 2$ + dual to conserved J_c on defect.

Would like to organize these into irreps of $OSp(4,4)$. Detailed decomp. of short $s_{max}=1$ irreps of $OSp(3,4)$ given by F. + Nicolai NPB 1984

Perfect match ~~match~~ if assume $SO(3)$ = diag. subgp. of $SU(2)_H \times SU(2)_V$

Nontrivial because bosonic content of irreps is 1 vector + 5 scalars of specific Δ and ℓ.

This is one of ~~that~~ D3-wrapped D5 brane systems in SUSY (with $\geq$ 6 supercharges.)

Spectrum contains 3 (stable) tachyons

$\Rightarrow$ 3 relevant deformations of dCFT.

VI The Field Theory

i) ambient $\mathbb{R}^4$ $\mathcal{N}=4$ SYM usual action

ii) $\mathbb{R}^3$ defect: hyperm. $q^i, \psi^i \in N$ of $SU(N)$

$i = 1,2$ of $SU(2) = \text{diag}$ of $SU(2)_H \times SU(2)_V$

A subset of 4d fields interacts on defect:

Strategy to find classical action: Hori 0012179

1) extend "superspace bdy" method

→ general action with 2 supchgs.

(i.e. $\mathcal{N}=1$ in $d=3$) and $SU(2)_{\text{diag}}$ sym.

2) impose $SU(2)_H \times SU(2)_V$ on component action

to achieve 8 supchgs ($\mathcal{N}=4$ in $d=3$).

A. Susy generators in $\mathcal{N}=1$ $d=4$ Majorana superspace: $S_\alpha = \frac{\partial}{\partial \bar\theta_\alpha} - i(\gamma^\mu \theta)_\alpha \frac{\partial}{\partial x^\mu}$

those containing $\frac{\partial}{\partial x^3}$ are broken by defect, the rest are preserved.

Hori: an efficient way to separate good + bad SUSY's and write actions inv. under the former.

i) Choose Majorana basis of γ^M so that

any spinor $\theta_\alpha \longrightarrow \theta_{\alpha(\tau)}$ $\quad \alpha = 1,2$ $\quad \tau = 1,2$

α is index on which $SO(2,1)$ acts.

ii) restrict superspace by $\theta_{\alpha(2)} = 0$

Then $S_{\alpha(1)}$ is indep of $\frac{\partial}{\partial x^2}$ and generates std $\mathcal{N}=1$ $d=3$ SUSY.

iii) take any 4d sf: $\Xi(x^M, \theta_\alpha)$

Define its restriction to "superspace bdy" by

$$\Xi(\underset{\sim}{y}, \theta_\alpha)\Big|_\partial = \Xi(\underset{\sim}{y}, x^3=0, \theta_{\alpha(1)}=\theta_\alpha, \theta_{\alpha(2)}=0)$$

↑ This acts on a std 3-d sf. and can be multiplied by any intrinsic 3d sf, such as

$$Q^i = q^i + \bar\theta \psi^i + \tfrac{1}{2}\bar\theta\theta f^i \quad \leftarrow \text{hyperm.}$$

For any 4d chiral sf: $\Phi(x,\theta)$

both $\mathrm{Re}\,\Phi\big|_\partial$ and $\mathrm{Im}\,\Phi\big|_\partial$ are 3d sf's.

Let $\Phi^A = (X_V^A + i X_H^A) + \bar\theta \chi^A + \frac{1}{2}\bar\theta\theta (F_V^A + i F_H^A)$

$X_V^A = X^{7,8,9}$ $\quad X_H^{3,4,5}$ $\quad A = 1,2,3$

↑ a chiral sf in $\mathcal{N}=4$ SYM thy.

Defect action involves:

$$\mathrm{Re}\,\Phi^A\big|_\partial = X_V^A + \bar\Theta\chi_{(1)}^A + \tfrac{1}{2}\bar\Theta\Theta(F_V^A - i D_2 X_H^A)$$

X_V^A, $\chi_{(1)}^A$ are main 4d fields which interact on defect (as do $A_{0,1,2}$, λ_1 from VMd but $D_3 X_H^A = \partial_3 X_H^A + g[A_3, X_H^A]$ also interact.

(ii) Defect action in $\mathcal{N}=1$ $d=3$ superspace:

$$S = \int d^3y\, d^2\Theta \left[\nabla_\alpha \bar Q^i \nabla_\alpha Q^i + g' \bar Q^i \sigma^A_{ij} Q^j \mathrm{Re}\,\Phi^A\big|_\partial \right]$$

$$\nabla_\alpha = D_\alpha - i g \Gamma_\alpha = D_\alpha - i g (D_{\alpha(3)} V)\big|_\partial$$

Invariant under: $\mathcal{N}=1$ $d=3$ SUSY $SU(N)$ gauge

$SU(2)_{\text{diag}} \times U(1)$ flavour sym ← on Q^i

B. Implement $SU(2)_H \times SU(2)_V$ in components:

eg $\mathcal{L}_{Yuk} = \bar{\psi}^i \left[i g \lambda \delta_{\dot{\alpha}j} + g' \sigma^A_{\dot{\alpha}j} \chi^A \right] q^j$

gaugino ↗ ↘ 3 chiral matter

To get full R-symmetry

i) set $g = g'$ — as in linear σ-model

ii) define bespinor $\lambda_{\dot{\alpha}j} = \lambda \delta_{\dot{\alpha}j} - i \sigma^A_{\dot{\alpha}j} \chi^A$

$$\mathcal{L}_{Yuk} = i g \, \bar{\psi}^i \lambda_{\dot{\alpha}j} q^j$$

↖ $(0, 1/2)$ $(1/2, 1/2)$ ↙ $(1/2, 0)$ of $SU(2)_H \times SU(2)_V$

Potential terms have manifest R sym after elimination of f^i, F^A:

Final defect action in components

$$S = \int d^2y \Big[D_k \bar{q}^+ D^k q - i \bar{\psi} \not{D} \psi \qquad \text{kinetic}$$

$$- g \bar{\psi} \sigma^A X_V^A \psi + i g \bar{\psi} \lambda q + \text{h.c.} \qquad \text{Yukawa}$$

$$- g \bar{q} \sigma^A D_3 X_H^A q + g^2 \varepsilon^{ABC} \bar{q} \sigma^A [X_H^B, X_H^C] q$$

$$+ \tfrac{1}{2} g^2 \bar{q} \{ X_V^A, X_V^A \} q - g^2 \delta(0) (\bar{q} \sigma^A q)^2 \Big] \qquad \text{potential}$$

$\delta(0)$ from elim of $F_V^A \sim X X^2 - \delta(x_3) \bar{q} \sigma^A q$

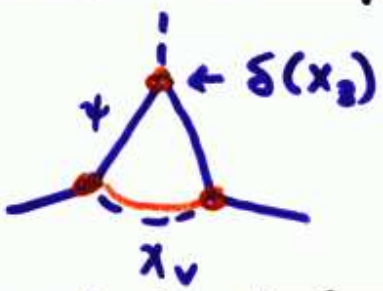
Artefact

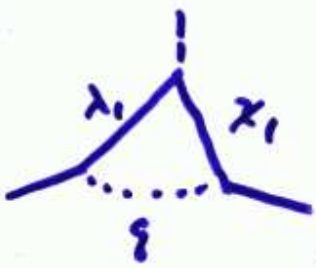
$= \dfrac{1}{(y - y')^4}$

Mirabelle Pestun 97

VII Perturbative Field Thy

A. Basics: 1-loop graph for $\langle X_v(x,\vec{y})\,\bar{\Psi}(y_1)\,\Psi(y_2)\rangle$





X_v pinned to defect

unpinned

Std 4d scalar prop: $\frac{1}{(x-x')^2+(\underset{\sim}{y}-\underset{\sim}{y}')^2} = FT_4\,\frac{1}{k^2+\underset{\sim}{k}^2}$

when pinned on both ends $\frac{1}{(\underset{\sim}{y}-\underset{\sim}{y}')^2} = FT_3\,\frac{1}{|\underset{\sim}{k}|}$

more singular than std 3d prop $\frac{1}{|\underset{\sim}{y}-\underset{\sim}{y}'|} = FT_3\,\frac{1}{\underset{\sim}{k}^2}$

For this simple reason, a 3d defect thy. with gauge + Yuk. couplings — normally superrenorm. — is promoted to critically renorm.

Canonically couplings g, g' run logarithmically, but with 8 supercharges, it might be conformal!

B. $\delta(x_3)$ gives clean separation of pinned + unpinned contribs. to correlators.

Gauge gp also gives separation:

i) unpinned graphs always occur with f^{abc} of non-abelian $SU(N)$ thy, so

ii) pinned graphs are whole story for $U(1)$ thy. (in which $n=4$ $d=4$ thy is free)

C. Detailed pwr ctg + sym. arg. for pinned graphs

i) only divergences in wave fn renorm of g^c, ψ^c. ← gauge dependent non-observable don't spoil conf. sym.

ii) g is not renormalized by defect interacts.
g' would be " if $g \neq g'$
but not if $g = g'$ by R-sym.

iii) no new divergent couplings by various symmetries.

A vigorous pf. that U(1) thy is a dCFT!

D. Possible problems in SU(N) thy from unpinned graphs.

more singular than pinned
non-trans inv in X.

Unsettled question ...

$\delta(0)(\bar{\xi}\sigma\xi)^2$ is a harmless artefact. It cancels a $\delta(0)$ from $\partial_3 \chi_H$ or change

VI Operator Matching: lowest KK mult. on gravity side contains

Spin	Δ	(l_H, l_V)	Operators
0	1	(1, 0)	$\bar{\xi}^i \sigma^A_{ij} \xi^j$
1/2	3/2	(1/2, 1/2)	$\bar{\xi}^i \psi^j$
0	2	(0, 1)	$\psi\sigma^A\psi + 2\bar{\xi}\chi^A_V\xi$
1	2	(0, 0)	$J_h = \bar{\psi}\rho_h\psi + i\bar{\xi}\overleftrightarrow{D}_h\xi$

Primary Op $\bar{\xi}\sigma^A\xi$ is unique op with $\Delta = 1 = l_H$ others found by action of bispinor supchg. Q^{ij}

This mult. contains conserved J_h on defect.

Comments:

1. $\Delta_0 = 1$ R-sym singlet $\bar{\xi}^i\xi^i$ is **not** in op. map

Expect $\Delta = 1 + a_1(g^2N) + \ldots$ ← not protected dual to str. state

A weak coupling calc. proves $a_1 \neq 0$

2. What ops. are dual to higher KK gravitons with $\Delta = \ell_H > 1$?

Candidates: i) highest $\ell_H = \ell$ components of

$$(\bar{\xi}\sigma^{A_1}\xi)\ldots\ldots(\bar{\xi}\sigma^{A_\ell}\xi) \quad *$$

ii) $\ell_H = \ell$ components of

$$(\bar{\xi}\sigma^{A}\chi_H^{B_1}\ldots\chi_H^{B_{\ell-1}}\xi) \quad \checkmark$$

Which is right?

i) intuitive arg that ops. should be formed from defect fields favors *

ii) a bronzy T-duality arg. favors $\chi_H^A = \chi^{3,4,5}$ ✓

Weak coupling calcs.

i) 1-loop calc shows that highest ℓ projection of * has no anom dim to order g^2N

ii) a more precise calc is req'd to decide the question for ✓. Not yet done.

Thus:

i) Field thy has natural candidate operators for duals to gravity fields

ii) Weak coupling calcs can illuminate situation.

Conclusion:

i) evidence for new type of dCFT, and

ii) extended AdS/CFT of KR

but ∃ open ?'s.